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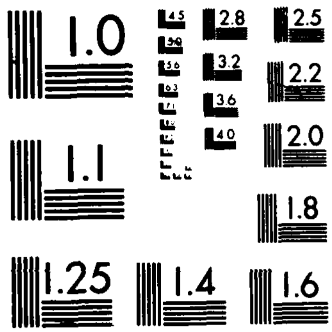

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TECHNICAL REPORT BRL-TR-2750

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THE FORWARD AND ADJOINT TRANSPORT OF BULLETS IN MONTE CARLO AIR-DEFENSE-END-GAME BALLISTIC VULNERABILITY PROBLEMS

William B. Beverly

July 1986

US ARMY BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

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Computerized Monte Carolo methodologies are routinely used to estimate the effectiveness of weapon systems. In particular, such codes are used at this installation to simulate the performance of air defense gun systems. In general, these methodologies are constructed to conduct sample firing events by picking values for the system uncertainties from their natural probability density functions. Therefore, such methodologies can be easily (Continued on next page)

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constructed with a logic which compares one-to-one with real-life firing events without ever constructing an explicit formulation of the associated expected-value-of-kill integrals. However, this ease of construction is not an unmitigated advantage since an analytical formulation may permit the development of more efficient procedures for calculating conventional kill probabilities as well as providing the basis for conducting more elaborate studies such as error and parameter-sensitivity studies. For example, in a "natural" Monte Carlo simulation, the fractional uncertainty in a kill estimate increases as an increasing fraction of the sample bullets miss the target. The efficiency of such a calculation may be improved if procedures could be developed where most of the sample bullets hit the target.

Therefore, our first objective in this report is to develop a mathematical formulation whose modeling of weapon-system uncertainties is similar to the current methodologies being used. To accomplish this, we formulate the expected-value-of-kill integral where the variables of integration are those used in the derivation of a forward, or "natural", Monte Carlo evaluation. Then, in an attempt to develop alternate formulation which may permit the more efficient calculation of kill probabilities, we transform the integrand to an adjoint form such that, in its Monte Carlo evaluation, the majority of the sample bullet trajectories will intersect the target. We then outline general procedures for conducting Monte Carlo evaluations of both the original and the transformed expected-value-of-kill integrals. Finally, we devise an example, two-dimensional problem, formulate the expected-value-of-kill integral to both the "natural" and the adjoint mode, and compare the efficiencies of the Monte Carlo evaluation of each mode for a series of different scenarios.

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I. INTRODUCTION

Ongoing work at the US Army Ballistic Research Laboratory (BRL) is aimed at predicting the effectiveness of air-defense gun systems. Computerized methodologies are used to solve representative end game problems. One such methodology, illustrated by the MGEM Monte Carlo computer program,[1] is currently used at BRL to calculate the expected value of kill for aircraft exposed to point detonation (PD) bullets fired from gun systems located on the ground. However, the procedures used in MGEM are not adequate for predicting the effectiveness of proximity fuzed (PX) bullets and need to be extended to obtain a methodology with such a capability. Additionally, the need for even more complex methodologies is anticipated in the future when the feasibility of using "smart" PX bullets, that is bullets whose trajectories can be changed during flight in order to take advantage of radar fixes of target location at times subsequent to the firing of the bullet, is investigated. The objectives of this study are to formalize the MGEM methodology, and to develop alternative Monte Carlo methods of solution which may be advantageously used in the future in the construction of PX and "smart-bullet" methodologies.

The expected effectiveness of a gun system depends upon the accumulation of the uncertainties associated with target detection and prediction of its future locations, the prediction of the bullet flight toward a target interception, and the incapacitation of the aircraft by a hit. For the purpose of our discussion, we have lumped these uncertainties into nine general categories which are tabulated below. In this tabulation and in the following discussion, we will use the standard conventions of probability as described by C. Eisenhart and M. Zelen[2] whereby a random variable is identified by using an upper case letter and its values are identified by using the associated lower case letter.

In no particular order, the uncertainties and the random variables used to define these uncertainties along with other necessary variables and notational conventions are tabulated below:

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[1]R.A. Scheder, "Modern Fire Control Analysis Using the Modern Gun Effectiveness Model (MGEM)," U.S. Army Materiel Systems Analysis Activity Technical Report No. 180, October 1976.

[2]C. Eisenhart, and M. Zelen, "Elements of Probability," Handbook of Physics, E.U. Condon, and H. Odishaw, Editors, McGraw-Hill Book Company, Inc., New York, 1958.

1. Variation between Rounds. One result of unavoidable differences between individual rounds of ammunition is a spread in the muzzle velocities V_m of the associated bullets.

2. Bore-Sight Error in the Gun. This error causes an uncertainty in the direction (elevation angle Θ_0 and azimuth angle Ψ_0) of the bullet as it leaves the gun. The subscript 0 is used here and elsewhere to identify variables associated with the birth of a bullet, taken to be the muzzle of the gun. Associated with these firing angles, we will also identify here the line-of-sight (LOS) range l which is taken to be the distance at some flight time t_f that a bullet would travel in still air with no gravity (Figure 1). The values of (Θ_0,Ψ_0) are assembled with $l(t_f)$ to obtain the LOS range vector $[\theta_0,\psi_0,l(t_f)]$ which is used to calculate the location of a bullet during its flyout.

The reader in referring to Reference 1 should note that our "l" is the same as his "r". We have reserved the bold face letter "**r**" with an appropriate subscript to identify position vectors, while as noted at the time of use, the non-bold face letter "r" is the range of the bullet in the spherical coordinate system of the gun.

Depending on analytical convenience, a bullet can be located in either a Gun Coordinate System (GCS) or a Target Coordinate System (TCS). The GCS is located with its origin at the center of the gun muzzle which is also taken to be the birth site of bullets. Depending on the analytical convenience of the moment, either spherical coordinates (θ,ψ,r) or Cartesian coordinates (x,y,z) will be used. The polar angle θ is measured from the positive z-axis and the azimuthal angle ψ is measured from the positive x-axis. In compact notation, a bullet is located in the GCS by the position vector $\mathbf{r}_b$ and the reference point in the target aircraft is located by the position vector $\mathbf{r}_a$. The TCS will be defined when needed in Section II of this report.

3. Dispersion between Shots in a Burst. The servo controlled return of the gun to a ready-to-fire state between shots in a burst is not precise. Consequently, an additional uncertainty in the initial direction of motion of a bullet is added to Item 2 where the uncertainty is expected to increase with successive firings in a burst.

4. Variation in Air Resistance to the Flight of a Bullet. Different orientations of a bullet with respect to its trajectory will cause the resistance by air to vary. This effect is quantified as an uncertainty in the drag coefficient C_d. In both MGEM and the present study, the uncertainty is taken to be Gaussian where C_d is assumed to have the same value over the

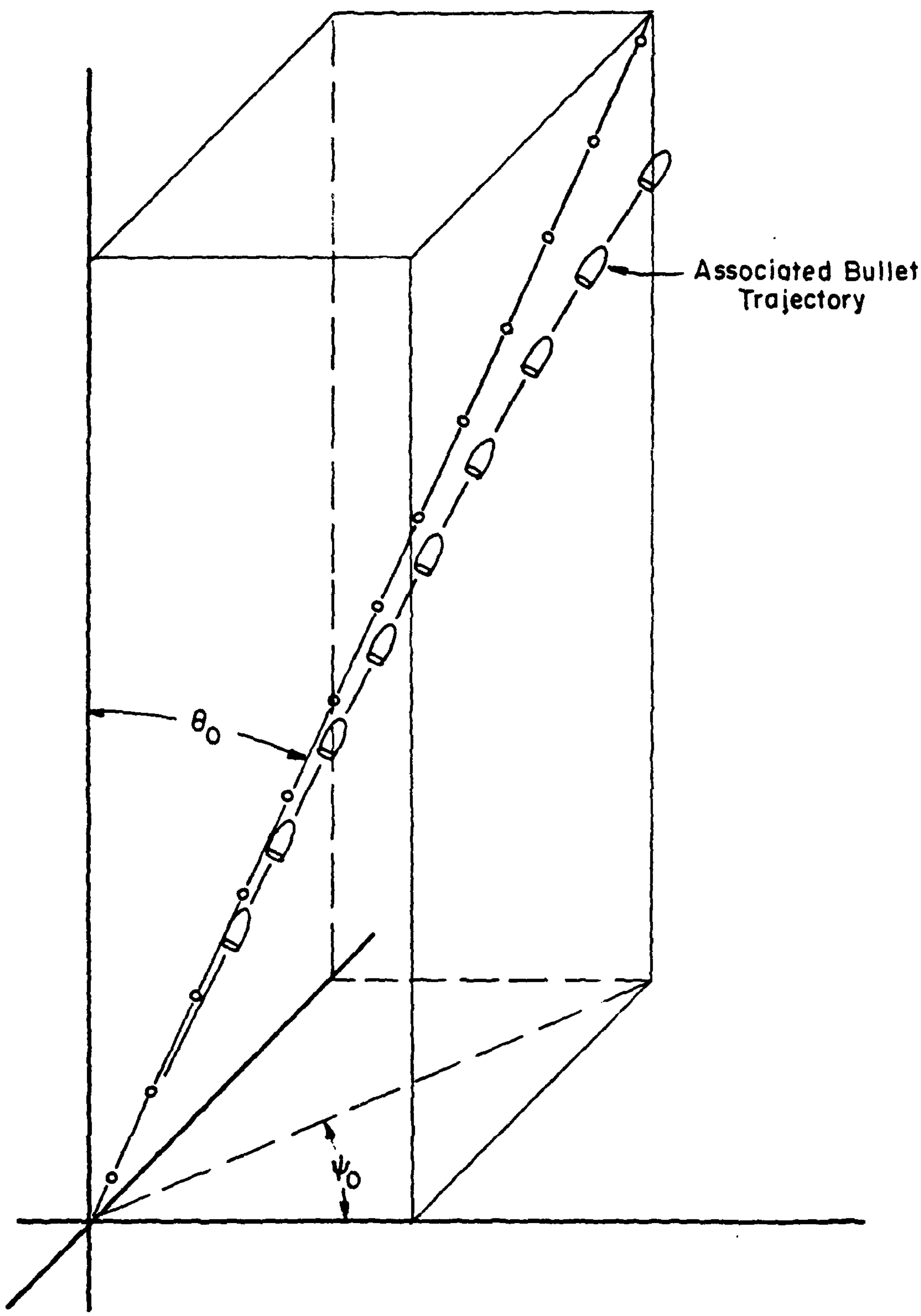

Figure 1. The Line-of-Sight (LOS) Range Vector for an Example Air-Defense End-Game Bullet Trajectory (No Wind Effects)

total trajectory of a bullet.

5. Variations in Wind. The effect of wind on a trajectory may differ from the wind corrections used to calculate aiming angles. This effect, (W_x and W_y), is assumed to be appreciable only in the horizontal plane. In both MGEM and the present study, this uncertainty is also taken to be Gaussian where W_x and W_y are taken to have the same sets of values over the total trajectory of a bullet.

6. Time of Firing. We allow in our formulation for an uncertainty in the time of firing T_0 although this random variable is assumed to be precisely known in the MGEM methodology.

7. Target Maneuver. The path of a target aircraft from the time of firing of a bullet to its anticipated interception by the bullet can not be predicted by the gun system with absolute certainty. In common with MGEM, we will assume that some method of optimal estimation such as the Kalman filter[3] is used to predict the most probable point of interception derived from earlier radar fixes of target location and velocity. In Monte Carlo calculations, a target maneuver is selected from a collection of standard maneuver paths where each path is composed of the aircraft position, velocity, acceleration, and orientation at regular time intervals, usually 0.05 seconds.

8. Errors in the Radar Fixing of a Target Location. The true location and velocity of a target aircraft may differ from that determined by a fire-control radar. This will lead to an additional uncertainty in the predicted future locations of the target with a resultant uncertainty in the calculated aiming angles for the gun. We identify the estimated aiming angles which are derived from an optimal estimation of future target locations based on a particular set of radar fixes as $(\overline{\Theta}_0,\overline{\Psi}_0)$. The values for this pair of random variables are taken to be the mean values of the firing angles described earlier in Item 2. In the present study, we assume that all of this uncertainty is acquired during the detection and analysis of the radar signals, and that the relative location of the radar in the GCS is precisely known.

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[3]A. Gelb, J.F. Kasper, Jr., R.A. Nash, Jr., C.F. Price, and A.A. Sutherland, Applied Optimal Estimation, A. Gelb, Editor, The M.I.T. Press, Massachusetts Institute of Technology, Cambridge, Massachusetts, and London, England, 1974.

9. The Kill Probability of a Critical Component in the Target Aircraft by a Bullet. In general, the causal prediction of some specified loss of capability by a component due to a bullet is a huge task which would require an impractical, if not impossible, detailed calculation of the bullet-target interaction. The description of all aspects of this phenomena would require a large number of random variables; for example, the generation of metal fragments from a detonating PX round and the penetration of these metal fragments into the target aircraft may require a number of random variables approaching Avagodro's number since the crystalline structure of the bullet and the target are involved. Instead, attempts have been made in ballistic-vulnerability methodologies to characterize the behavior of a large aggregate of similar rounds of a type by a small, manageable number of random variables and hope that the true expected-value-of-capability loss is adequately approximated by a function of these used random variables. For example, according to Kruse and Brizzolara[4], the $P_{K|H}$ (probability of a kill given a hit) function for a component is generally given in terms of the mass and velocity of a metal fragment as it impacts the outer surface of a critical component. An attempt to improve the mass-velocity characterization of metal fragments is also mentioned in this reference.

Item 9 completes the tabulation of the lumped uncertainties associated with air-defense end-game problems. In general, the probability density of random-variable values associated with an uncertainty is not known, but in practice is assumed to be adequately approximated by some probability density function which is tractable to mathematical analysis. In the Monte Carlo simulation of engagements by the current MGEM, uncertainties Nos. 1-5 are assumed to be described by independent Gaussian distributions. We include uncertainty No. 6 in the tabulation even though the firing time is assumed to be precisely executed in the present version of MGEM. Such a causal (or unitary) PDF is described in the mathematical formulation by a Dirac delta function (Reference 2). Uncertainty No. 7 is usually taken into account in end-game methodologies by using prediction algorithms based on noisy estimates of the target state. These latter result from uncertainty No. 8, and are generated in the code by imposing appropriate sensor errors on a predetermined precisely known flight path which is representative of a selected aircraft maneuver.

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[4]L. Kruse, and P. Brizzolara, "An Analytical Model for Deriving Conditional Probabilities of Kill for Target Components," U.S. Army Ballistic Research Laboratories Report No. 1563, December, 1971, (UNCLASSIFIED).

We will now outline a brief summary of a typical MGEM calculation where the firing doctrine for a total engagement with a target aircraft is simulated. This outline is non-mathematical and is used to introduce the random variables in the context of which they will be used later to construct the expected-value-of-kill integrals for a selected set of conditions in an engagement.

1. Select the type of aircraft maneuver for which the expected value of kill is desired, and retrieve the predetermined flight path for this type of maneuver.

2. Simulate radar fixes on the target aircraft at selected times. The location, velocity, and acceleration of the target are either measured or calculated for these times.

3. Using the data derived in Step 2, calculate an optimal prediction of future target locations. Based on the chosen firing doctrine, select a value for the firing time T_0 of the first bullet in the engagement.

4. The location $\mathbf{r}_b$ of a bullet at any time t is assumed to be adequately approximated by the bullet flyout equations given in the next section of this report. Using the data derived in Step 3 and the mean values of V_m, C_d, W_x, W_y, and T_0, determine an estimated interception point $(\mathbf{r}_b)_e$ and interception time t_e for the target aircraft by the bullet. Values for the polar aiming angles $(\overline{\Theta}_0,\overline{\Psi}_0)$ are also derived in this calculation.

5. Pick a set of values for the initial direction (Θ_0,Ψ_0) of the bullet as it leaves the gun by sampling the PDF which describes the uncertainties associated with implementing the aiming angles calculated in Step 4. In MGEM, this PDF is taken to be Gaussian where the values of the aiming angles $(\overline{\Theta}_0,\overline{\Psi}_0)$ are taken to be the mean values of (Θ_0,Ψ_0). Additionally, an error bias is added which systematically increases the aiming angle uncertainty for later bullets in a burst. These sample values are identified as $[(\theta_0)_{ij},(\psi_0)_{ij}]$ where i is the running index over bullets in an engagement and j is the running index over engagements. The number of bullets fired in an engagement is taken to be I and the number of engagement replications in the Monte Carlo calculation is taken to be J.

6. Pick a value for each of the random variables (V_m,C_d,W_x,W_y) by sampling the appropriate Gaussian PDF. These sample values are identified as $[(v_m)_{ij},(c_d)_{ij},(w_x)_{ij},(w_y)_{ij}]$. These values are used below to calculate the necessary points on the trajectory of the sample bullet.

7. Simulate the firing of the sample bullet and calculate its true location $(\mathbf{r}_b)_e$ at the predicted time of interception t_e. The location of the bullet which has been in flight for a time $(t_f)_e$ is calculated by applying a differential correction for the effect of gravity and wind to the line of sight (LOS) range vector $[(\theta_0)_{ij},(\psi_0)_{ij},\ell]$ (Figures 1 and 2).

8. Determine the true location $(\mathbf{r}_a)_e$ of a reference point in the target aircraft at time t_e and calculate the distance between $(\mathbf{r}_a)_e$ and $(\mathbf{r}_b)_e$.

9. Calculate the distance between the bullet and the target at intervals of Δt about t_e until a minimum distance is obtained.

10. Using the two calculated distances adjacent to the calculated minimum distance, interpolate to obtain an approximation of the distance of closest approach (miss distance) between the target and the bullet. The location of the bullet at its point of closest approach is identified as $(\mathbf{r}_b)_m$ and the associated time is identified as t_m.

11. The orientation of the target aircraft in the GCS can be fixed by either locating three points (the aforementioned reference point plus two additional points in the aircraft), or by two vectors emanating from the reference point (Figure 2). Without further specification, identify these two vectors as ω_a. Determine the orientation $(\omega_a)_m$ of the target at time t_m. Calculate the area of the target presented to the incoming bullet. Representing this presented area as a circle, determine if the bullet impacts the target. Go to Step 14 if the bullet misses the target.

12. Determine if the aircraft is killed by comparing a random number with the ratio of the vulnerability area to the presented area of the aircraft.

13. Provision is also made to treat multiply-vulnerable components, the destruction of any one of which will not by itself result in target kill. If single-hit kill is not found in Step 12, determine by random number draw whether one of these components was destroyed and whether this in conjunction with previous damage results in target kill.

14. Reiterate Steps 1-13 for the remaining bullets in the burst as dictated by the firing doctrine. If target kill is achieved, terminate the engagement. Otherwise, reiterate burst firing until the firing doctrine dictates engagement termination.

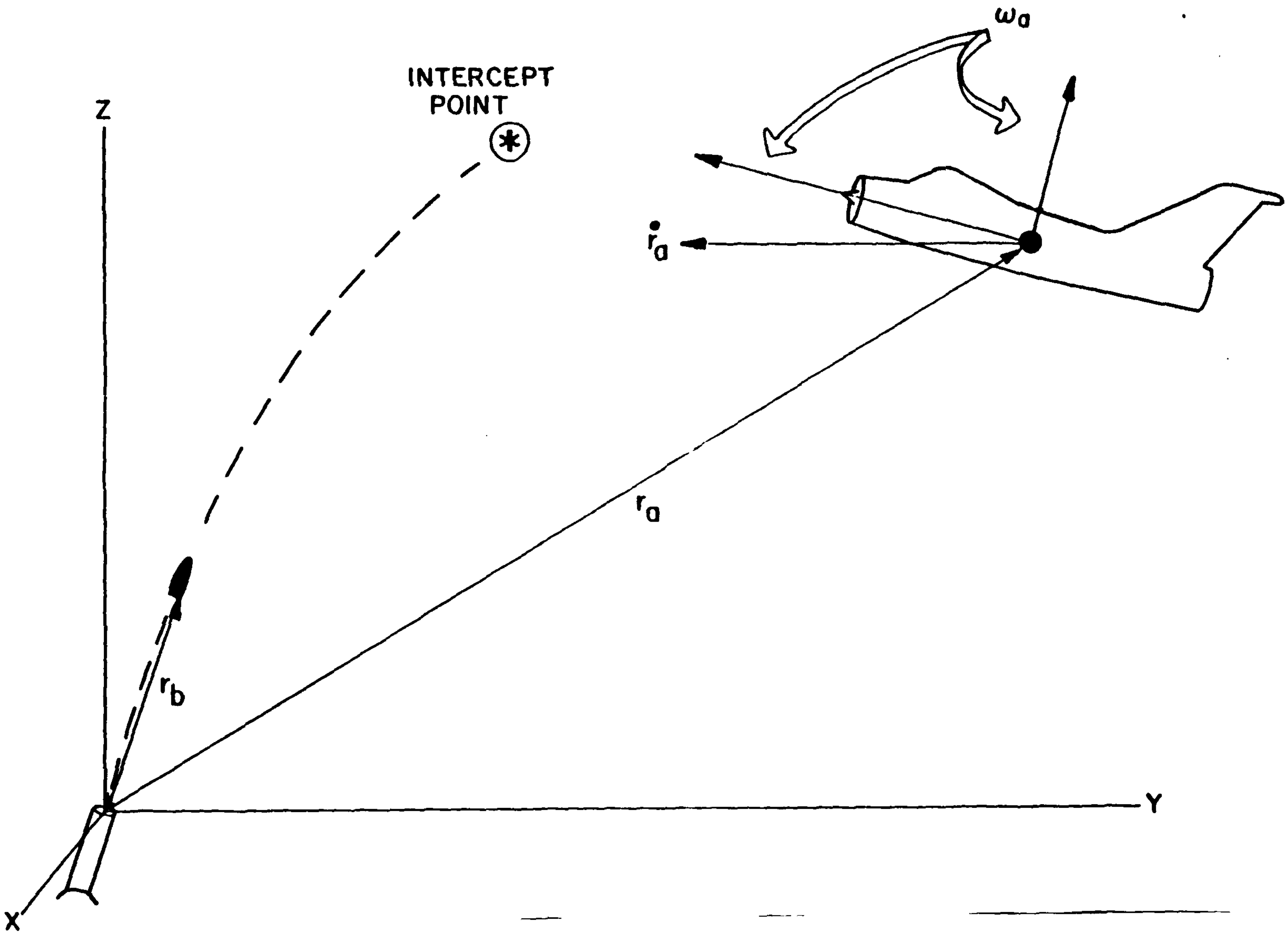

Figure 2. The Vectors (Gun-Coordinate System) Associated with an Air-Defense End-Game Problem

15. Reiterate Steps 1-14 until a total of J replications of the engagement have been obtained.

16. Calculate the expected value of kill $E\{K\}$ for the engagement as the average kill probability of the J replications, that is

$$E\{K\} = \frac{\textit{Number of Kills}}{\textit{Number of Replications}}. \tag{1}$$

Step 16 completes the outline of a typical Monte Carlo calculation by MGEM. We have discussed it in some detail to provide a concrete example of Monte Carlo methodology, before turning to more general mathematical formulations. In the next section of this report, we will first use the uncertainties tabulated earlier in this section to construct the expected-value-of-kill integral for a single critical component for a single set of firing conditions in an engagement. We will then restructure the integrand in this integral by changing the variables of integration (θ_0, ψ_0, e_d) to a new set which permits the picking of sample bullets which will with a higher probability interact with the target aircraft. Then, in Section III, we will outline in turn Monte Carlo procedures which can be used to calculate a forward estimate of the original "natural" expected-value-of-kill integral and an adjoint, or backward, estimate of its restructured form. The term "forward estimate" is used here to describe a calculation where each sample bullet is picked at the gun and then transported toward the target while the term "adjoint or backward estimate" is used to describe a calculation where each sample bullet is picked at a point on its trajectory near its target and then transported back to the gun. Finally, in Section IV, we devise an example, two-dimensional problem and outline its solution by using, in turn, forward and adjoint Monte Carlo sampling. The calculational efficiency of forward vs. adjoint estimation is compared for different target sizes. A simplified outline of the Monte Carlo evaluation of expected-value integrals is given in the appendix at the end of this report, and should be read before proceeding to the next section.

II. THE CONSTRUCTION OF EXPECTED-VALUE-OF-KILL INTEGRALS

Our main interest in this study is to pick sample bullets for Monte Carlo simulation calculations which have a high probability of interacting with the target aircraft. This interaction for a PD bullet will require a direct hit on the target, while a possible interaction for a PX bullet will merely require that the bullet pass into the fuzing region which surrounds the target. We will first formulate a "natural" form of the expected-value-of-kill integral and then change three of its variables of integration so that the transformed (adjoint) integral is in a form which is amenable to picking sample bullets which will interact with a high probability with the target.

Since our principal interest here is the development of alternate formulations of the expected-value-of-kill integrals and a comparison of the expected efficiencies of the Monte Carlo evaluation of these different formulations, we will not describe the detail associated with locating a target and firing a bullet to obtain its kill, but will merely identify the various associated uncertainties and represent them by probability density functions (PDF's). The general form of the expected value of kill $E\{K\}$ of a critical component in a target aircraft by a bullet fired from a gun at time t_0 is given by the integral of the product of the various PDF's used to predict the uncertainties associated with locating the target, aiming the gun, firing the bullet, transporting the bullet toward its aircraft target, and a kill function which approximates the effect of a bullet on the operation of a critical component in the aircraft. In order to provide a similar form for both the natural and adjoint formulation of both PD and PX munitions, we define a region R_f which for a PD bullet barely encloses the volume of the target and for a PX bullet barely encloses the volume of the target plus the associated region about the target in which a fuzing action by the bullet can occur. We will choose a sphere which seems to be the most convenient form for calculations (Figure 3). In each case, an associated surface S_f is identified which is that part of the boundary of R_f which can be intersected by bullet trajectories.

We assume that the lumping of the various uncertainties associated with an air-defense system into the form described earlier in Section I of this report will permit the expected-value-of-kill integral to be formulated as

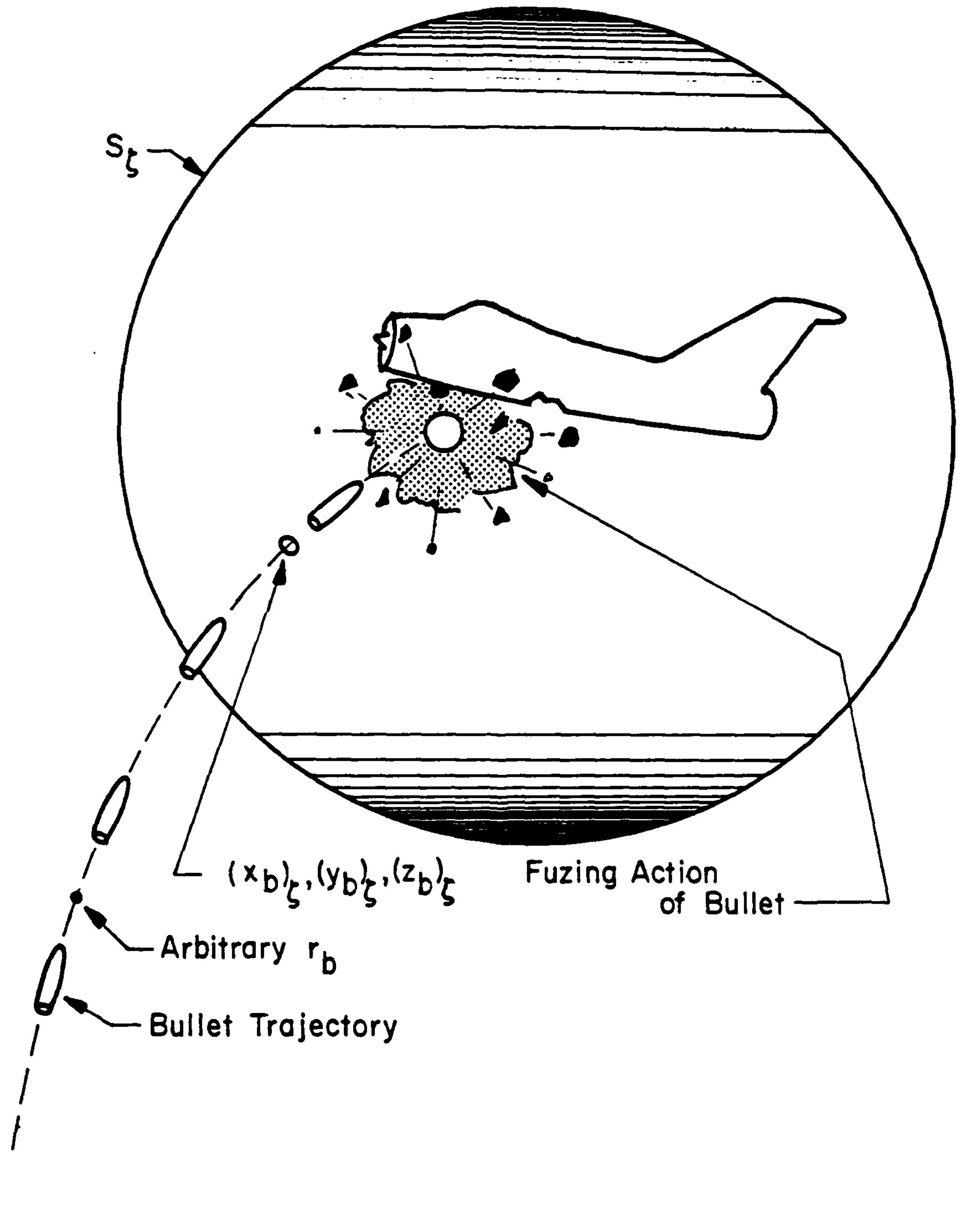

Figure 3. An Illustration of a Bullet-Trajectory Impact with an Example Surface S_ζ

$$E\{K(T_0,\overline{\Theta}_0,\overline{\Psi}_0,\Theta_0,\Psi_0,V_m,C_d,W_x,W_y)\} =$$

$$\int \cdot \underset{R_H^G}{\cdot} \cdot \int A(t_0) B(\overline{\theta}_0,\overline{\psi}_0) C(\theta_0,\psi_0|\overline{\theta}_0,\overline{\psi}_0)\ Sin\theta_0\ F(v_m)\ G(c_d)\ U(w_x,w_y)\ \times$$

$$Q_\varsigma^G[(\mathbf{r}_a)_\varsigma,(\dot{\mathbf{r}}_a)_\varsigma,(\omega_a)_\varsigma,(\mathbf{r}_b)_\varsigma,(\dot{\mathbf{r}}_b)_\varsigma,(\omega_b)_\varsigma]\ dw_x\ dw_y\ dv_m\ dc_d\ d\theta_0\ d\psi_0\ d\overline{\theta}_0\ d\overline{\psi}_0\ dt_0\ . \qquad (2)$$

The quantity R_H^G is the region in $(t_0,\overline{\theta}_0,\overline{\psi}_0,\theta_0,\psi_0,v_m,c_d,w_x,w_y)$ space which includes all possible firing events where the bullet trajectories will intersect S_ς. In practice, since the commonly used Gaussian PDF's are truncated so as to include only a reasonable region about their mean value, we will take R_H^G to include only those points where all of the various PDF's are non-zero.

This integral can be evaluated when the physics describing the bullet flyout is sufficiently well understood so that a description of all bullet-trajectory intersections of S_ς is calculable to an adequate accuracy. For example, in an end game methodology developed by Ford[5], the bullet flyout to target impact is calculated by using iterative procedures to solve the differential equation constructed from the forces acting on a bullet in flight. In the present study, after defining the quantities used above, we will apply the analytic functions used by Reference 1 to predict the flyout of bullets.

The order of integration of the integrals used in this study is assumed to start at the innermost integral and proceed toward the outer integrals. In general, we express the arguments of functions in terms of their most convenient variable association without regard to whether the variables used are variables of integration. Needless to say, before conducting an integration, all functions must be given in terms of the variables of integration.

In Equation 2, we have allocated the representation of the various uncertainties to six probability density functions where each function describes a particular aspect of the total problem. The uncertainties associated with the firing time are described by the PDF $A(t_0)$. If, as presently assumed in MGEM, a precisely known firing time is assumed, its PDF is a Dirac delta function, that is

$$A(t_0)=\delta(t_0-b), \qquad (3A)$$

[5]"DIVAD Contract Data Requirement No. A00101, System Simulation Analyst Manual, Revision A," Ford Aerospace and Communication Corporation, Aeronutronic Division, Newport Beach, California 92663, November 1980, (Competition Sensitive).

where t_0 is the precisely known firing time. The remaining temporal variables, which will be explicitly used in the adjoint formulation later in this section, are related by

$$t = t_0 + t_f, \tag{3B}$$

where t_f is the time of flight of a bullet and t is the target time. For a given value of T_0, the associated infinitesimal times are related by

$$dt_f = dt. \tag{3C}$$

The uncertainties associated with locating the target, predicting its future locations, and calculating the aiming angles for the gun are described by the joint PDF $B(\bar{\theta}_0,\bar{\psi}_0)$. In practice, the calculated values $(\bar{\theta}_0,\bar{\psi}_0)$ of the aiming angles are derived from an optimal estimate of future target locations based on a sequence of radar fixes obtained prior to the firing of the bullet. An uncertainty is associated with each radar fix so that an estimate of future locations of the same target derived individually from two alternate sequences of radar fixes will not in general be the same. Consequently, $(\bar{\theta}_0,\bar{\psi}_0)$ is generally some function of the radar uncertainties and the target maneuver whose analytic form would probably be very difficult to derive and very cumbersome to write.

In addition to the aiming uncertainties associated with $(\bar{\theta}_0,\bar{\psi}_0)$, the accuracy of the true firing direction (θ_0,ψ_0) of bullets is further degraded by uncertainties associated with positioning the gun for firing. We assume here that the conditional probability density $C(\theta_0,\psi_0|\bar{\theta}_0,\bar{\psi}_0)$ of real firing angles is the probability density per unit solid angle Ω. If the air-defense system has a non-zero probability for firing into the upper hemisphere (atmosphere) and is zero elsewhere, then its normalization is given by

$$\int_{2\pi} C(\theta_0,\psi_0|\bar{\theta}_0,\bar{\psi}_0)\,d\Omega = \int_0^{2\pi}\int_0^{\frac{\pi}{2}} C(\theta_0,\psi_0|\bar{\theta}_0,\bar{\psi}_0)\ Sin\theta_0\ d\theta_0\ d\psi_0 = 1, \tag{3D}$$

for any given set of values for $(\bar{\theta}_0,\bar{\psi}_0)$. For example, a constant value of $1/2\pi$ for $C(\theta_0,\psi_0|\bar{\theta}_0,\bar{\psi}_0)$ would correspond to firing isotropically into the upper hemisphere.

The variation in muzzle velocities among the bullets is described by the PDF $F(v_m)$. The value for the drag coefficient C_d is taken to be constant during the flyout of a bullet, and its values for an aggregate of bullets is predicted by the PDF $G(c_d)$. The values for the components of force due to

wind are predicted by the joint PDF $U(w_x,w_y)$ where U in MGEM[1] is assumed to be separable into the two functions

$$U(w_x,w_y) = f_1(w_x)\, f_2(w_y). \tag{3E}$$

A set of values when picked for W_x and W_y is also taken to remain constant for each bullet during its flyout.

As noted earlier[4], the loss of capability by a target component due to the interaction of a bullet is generally approximated as a kill-probability function of certain random variables which can be used to characterize an aggregate of bullets or metal fragments as they impact the outer surface of a critical component. The formulation used in Equation 2 differs from the definition given by Reference 4 in that the arguments of $Q_\varsigma^G[(\mathbf{r}_a)_\varsigma,(\dot{\mathbf{r}}_a)_\varsigma,(\omega_a)_\varsigma,(\mathbf{r}_b)_\varsigma,(\dot{\mathbf{r}}_b)_\varsigma,(\omega_b)_\varsigma]$ are the values in the GCS of the random variables at the time when a bullet or fragment impacts a hypothetical surface S_ς instead of the real exterior surface of a target component. As noted earlier in Section I, the position vectors $\mathbf{r}_a$ and $\mathbf{r}_b$ are used to locate a reference point in the target aircraft and bullet, respectively, and the vectors $\dot{\mathbf{r}}_a$ and $\dot{\mathbf{r}}_b$ are the velocities of these reference points in the target and bullet, respectively. The vector pairs, ω_a and ω_b, define the orientation of the bullet and target, respectively, where all vectors are defined in the GCS. The subscript ς is used to identify the associated vector at the intersection of the bullet with S_ς.

The quantity Q_ς^G is itself the expected value of kill of the bullet at its intersection of S_ς, and, in principle, assuming that the location and velocity vectors provide an adequate characterization in a stochastic formulation of bullet flyout, could have been described in Equation 2 by increasing the dimensionality of the multiple integrals to include the appropriate functions which describe the further transport of the bullet into ς and its interaction with the target aircraft. In practice during current Monte Carlo calculations for PD bullets, the kill function is approximated at the surface of the aircraft for damage due to both the penetration of detonation fragments into the target and the transmission of shock to components distant from the point of impact. In practice for a PX bullet in forward Monte Carlo calculations, an estimate of Q_ς^G would be obtained for each firing event by transporting the bullet into R_ς, simulating its detonation or non-detonation, and then deriving the kill probability from a kill function such as that described in Reference 4.

We will assume here that the mathematical relations used by Reference 1 can be used to adequately predict the flight of bullets toward a target aircraft. According to Reference 1, McShane, Kelley, and Reno[6] have derived the LOS range l to be given by the Siacci formula,

$$l = \frac{v_m t_f}{1+c_d\sqrt{v_m}\, t_f}, \tag{4A}$$

where t_f is the time that a bullet has been in flight. The effect of wind and gravity are assumed to be differential corrections to the predicted LOS range vector (θ_0,ψ_0,l) which are given in Cartesian coordinates by:

$$\Delta x = w_x\left[t_f - \frac{l}{v_m}\right], \tag{4B}$$

$$\Delta y = w_y\left[t_f - \frac{l}{v_m}\right], \tag{4C}$$

$$\Delta z = -gt_f^2\left[\frac{3+c_d\sqrt{v_m}\, t_f}{6(1+c_d\sqrt{v_m}\, t_f)}\right]. \tag{4D}$$

The quantities, Δx, Δy, and Δz, are the differential corrections, and g is the acceleration due to gravity.

As outlined in the next section of this report, a forward, or "natural", Monte Carlo evaluation of the multiple integrals in Equation 2 can be conducted by picking values for each of the random variables of integration by sampling the appropriate PDF's to obtain sample bullet trajectories. Each sample bullet is then transported along its trajectory using the preceding relations to determine if it interacts with the target aircraft. The effect of each interaction is determined and the expected value of kill is calculated as the mean of a large number of similar events. However, using calculational procedures such as these for cases where the target is at a large distance from the gun, a large fraction of the bullets will miss the target and contribute only zeroes to the expected value of kill probability. Consequently, the efficiency of the forward mode decreases as the hit probability decreases until, for very low hit probabilities, a prohibitive number of sample histories would be required to obtain an acceptable fractional

[6]E.J. McShane, J.L. Kelley, and F.V. Reno, Exterior Ballistics, University of Denver Press, 1953.

uncertainty $\frac{\delta \bar{P}_H}{\bar{P}_H}$ (where $\delta \bar{P}_H$ is the standard deviation of a hit-probability estimate $\bar{P}_H$).

We will now attempt to transform the "natural" expected-value-of-kill integral (Equation 2) to a form such that only bullets which have a high probability of interacting with the target can be picked. As a first step, we note that the region R_ς is the smallest sphere which just encloses the target aircraft when the bullets are PD and which just encloses the target aircraft plus its associated fuzing region when the bullets are PX. The radius of the sphere in each case is taken to be the constant a. We construct a Target Coordinate System (TCS) (Figure 4) whose origin is at the center of R_ς and whose orientation is such that the Cartesian coordinates of each point in the TCS is related to the same point in the GCS by a simple, time-dependent translation of the center (x_a, y_a, z_a) of R_ς in the GCS; that is, the (x', y', z') axes in the TCS are parallel to and have the same orientation as the (x, y, z) axes in the GCS (Figure 5). Similarly, the spherical coordinates (ϕ, ξ, s) in the TCS are similar to and have the same orientation as the spherical coordinates (θ, ψ, r) which are used in the GCS. The surface S_ς which bounds R_ς is in the TCS just that part of $s = a$ which can be impacted by bullets fired at the origin of the GCS.

We next note that a large fraction of the bullet trajectories which intersect S_ς will interact with the target aircraft; in fact, this fraction is relatively insensitive to factors affecting the hit probability such as target size and range. If we could change some of the variables of integration used in Equation 2 to a new set of variables such that all sample trajectories will intersect S_ς, then we should expect that more efficient calculations of estimates than are currently being made could be developed for air-defense problems where the expected value of hit is small.

To accomplish this change of variables, we note that the location $[(x_b)_\varsigma, (y_b)_\varsigma, (z_b)_\varsigma]$ in the GCS of the bullet intersection of S_ς is related to its location $[(x_b')_\varsigma, (y_b')_\varsigma, (z_b')_\varsigma]$ in the TCS by:

$$(x_b)_\varsigma = (x_b')_\varsigma + x_a(t), \tag{5A}$$

$$(y_b)_\varsigma = (y_b')_\varsigma + y_a(t), \tag{5B}$$

$$(z_b)_\varsigma = (z_b')_\varsigma + z_a(t), \tag{5C}$$

where $[x_a(t), y_a(t), z_a(t)]$ is the time-dependent location of the center of R_ς in the GCS. These relations become in spherical coordinates:

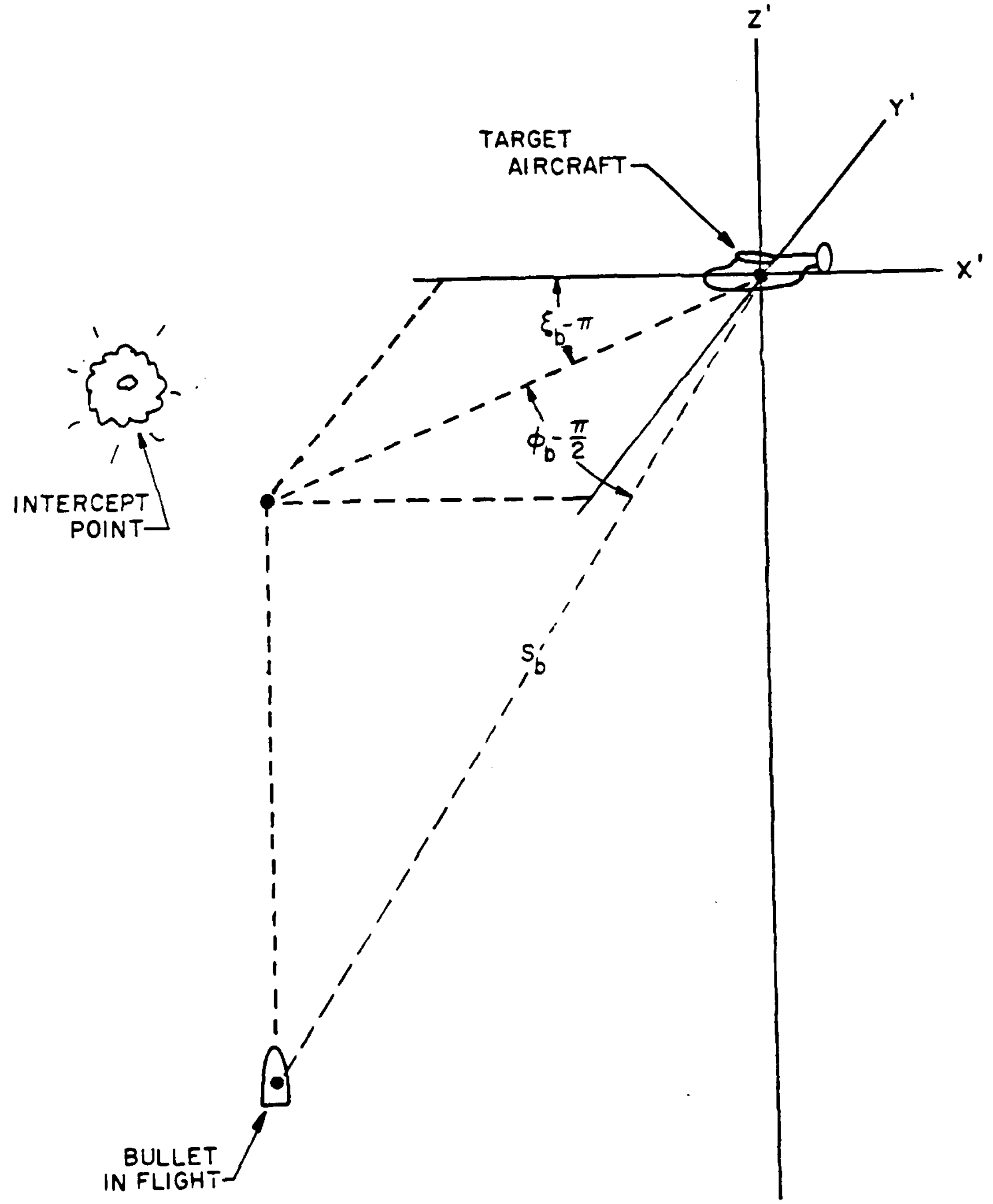

Figure 4. The Target Spherical Coordinate System Associated with the Change of Integration Variables

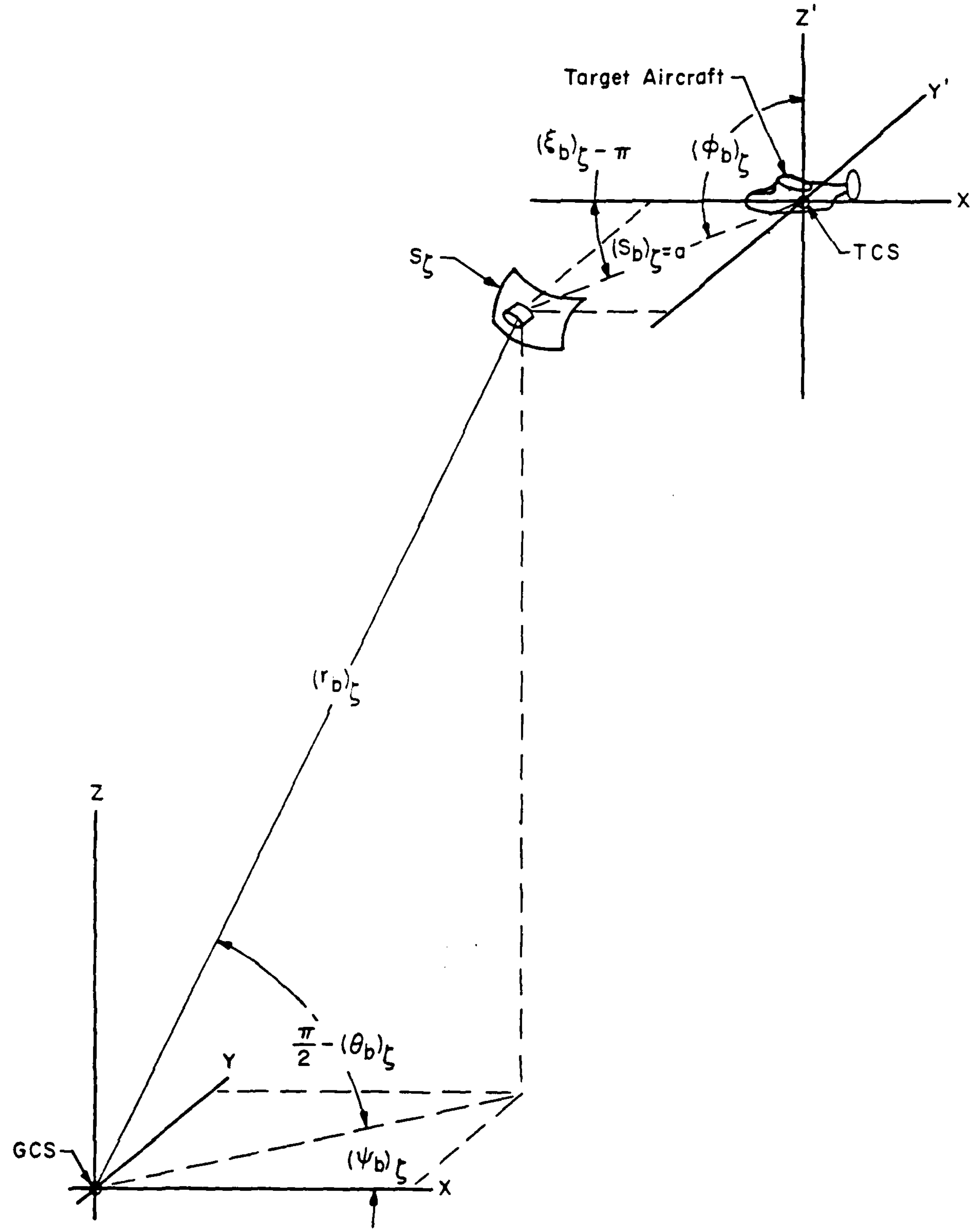

Figure 5. The Orientation of the Gun-Coordinate System and the Target-Coordinate System When the Bullet Intersects $S_\zeta\,(\mathbf{r} = (\mathbf{r}_b)_\zeta)$

$$l_\varsigma \sin\theta_0 \cos\psi_0 + w_x\left[(t_f)_\varsigma - \frac{l_\varsigma}{v_m}\right] = a\sin\phi_b\cos\xi_b + x_a(t), \tag{6A}$$

$$l_\varsigma \sin\theta_0 \sin\psi_0 + w_y\left[(t_f)_\varsigma - \frac{l_\varsigma}{v_m}\right] = a\sin\phi_b\sin\xi_b + y_a(t), \tag{6B}$$

$$l_\varsigma \cos\theta_0 - \frac{g(t_f)_\varsigma^{\,2}}{6}\left[\frac{3 + c_d\,(t_f)_\varsigma\sqrt{v_m}}{1 + c_d\,(t_f)_\varsigma\sqrt{v_m}}\right] = a\cos\phi_b + z_a(t). \tag{6C}$$

The quantities l_ς and $(t_f)_\varsigma$ are the LOS range and the bullet time-of-flight, respectively, for trajectory intersections of S_ς. Further noting that l_ς is given by the Siacci equation,

$$l_\varsigma = \frac{v_m\,(t_f)_\varsigma}{1 + c_d\,(t_f)_\varsigma\sqrt{v_m}}\ , \tag{4A}$$

and the temporal variables are related by,

$$t = t_0 + (t_f)_\varsigma\ , \tag{3B}$$

Equations 6A-6C provide three independent equations relating three of the original variables of integration, namely the spatial variables (θ_0,ψ_0) and either v_m or c_d, and the TCS spatial variables (ϕ,ξ) and the temporal variable t.

We will arbitrarily select c_d to participate in the transformation so that the sought change of variables is from (θ_0,ψ_0,c_d) to (ϕ,ξ,t). Assuming that all points in R_H^G will transform one-to-one to a region R_H^T in $(t_0,\bar\theta_0,\bar\psi_0,\phi,\xi,v_m,t,w_x,w_y)$ space, we can write the transformed expected-value-of-kill integral as

$$E\{K(T_0,\bar\Theta_0,\bar\Psi_0,\Theta_0,\Psi_0,V_m,C_d,W_x,W_y)\} =$$

$$E\{K[T_0,\bar\Theta_0,\bar\Psi_0,\Phi,\Xi,T,V_m,W_x,W_y]\} =$$

$$\int \bullet \underset{R_H^T}{\bullet} \bullet \int A(t_0)\, B(\bar\theta_0,\bar\psi_0)\, C(\theta_0,\psi_0|\bar\theta_0,\bar\psi_0)\, Sin\ \theta_0\, F(v_m)\, G(c_d)\, U(w_x,w_y)\ \times$$
$$J_\varsigma(\phi,\xi,t)\ Q_\varsigma^T[(\mathbf{r}_t)_\varsigma,(\dot{\mathbf{r}}_t)_\varsigma,(\omega_t)_\varsigma]\ dw_x\, dw_y\, dv\, dv_m\, dt\, d\phi\, d\xi\, d\bar\theta_0\, d\bar\psi_0\, dt_0\ . \tag{7A}$$

In this formulation, we have assumed that the component kill function $Q_\varsigma^{\,T}$ is given in the TCS. The quantity T is taken to be the random variable whose values are t. The Jacobian $J_\varsigma(\phi,\xi,t)$ is the absolute value of the determinant of the indicated partial derivatives at the time when the bullet trajectory intersects S_ς, that is

$$J_\varsigma(\phi,\xi,t) = \left|\frac{\partial(\theta_0,\psi_0,c_d)}{\partial(\phi,\xi,t)}\right| = \left|\begin{matrix} \frac{\partial\theta_0}{\partial\phi} & \frac{\partial\theta_0}{\partial\xi} & \frac{\partial\theta_0}{\partial t} \\ \frac{\partial\psi_0}{\partial\phi} & \frac{\partial\psi_0}{\partial\xi} & \frac{\partial\psi_0}{\partial t} \\ \frac{\partial c_d}{\partial\phi} & \frac{\partial c_d}{\partial\xi} & \frac{\partial c_d}{\partial t} \end{matrix}\right|. \tag{7B}$$

As is always the case, all functions in the integrand must be given in terms of the variables of integration before conducting an integration.

Equation 7A is in a form where our sampling objectives can be attained. A point on S_ς at some particular target time in conjunction with the known location of the gun and a set of sample values for the remaining variables of integration, unchanged from the forward formulation (Equation 2), will completely determine a bullet trajectory. A Monte Carlo evaluation will be developed in the next section of this report where sample trajectories are picked using these changed variables of integration.

However, we will note here that the calculational efficiency may be improved by picking the sample points $(\mathbf{r}_b)_\varsigma$ directly from S_ς instead of first picking values for the angles (Φ,Ξ) and then locating the associated point on S_ς. The relevant quantities are related by

$$dS_\varsigma = a \sin\phi \; d\phi \; d\xi. \tag{8}$$

Procedures for conducting such sampling as well as an analysis of their calculational efficiency should be investigated. However, we will not expand further on this line of development in the present study.

III. A GENERAL MONTE CARLO EVALUATION OF THE NATURAL AND THE TRANSFORMED EXPECTED-VALUE-OF-KILL INTEGRAL

A Monte Carlo procedure for evaluating the multiple integrals in equation 2 is outlined below and illustrated in Figure 6. In order to keep the following outline as general as possible, we will neither describe the sampling procedures used to pick variable values from the various PDF's nor attempt to develop optimum procedures for picking values for the new variables (t,ϕ,ξ). This procedure is composed of the following steps:

1. Select the type of aircraft maneuver for which the expected value of kill is desired, and retrieve the predetermined flight path for this type of maneuver.

2. Simulate radar fixes on the target aircraft at selected times. The location, velocity, and acceleration of the target are either measured or calculated for these times.

3. Use the data just derived in Step 2 to calculate an optimal prediction of future target locations. Applying some preselected firing doctrine, pick a value for the firing time T_0 by sampling $A(t_0)$. This sample firing time is identified as $(t_0)_i$.

4. The location $\mathbf{r}_b$ of a bullet at time t is assumed to be adequately approximated by the flyout relations (Equations 4A-4D) given in Section II of this report. Use the data derived in Step 3 and the mean values of (V_m, C_d, W_x, W_y) to determine an estimated interception point $(\mathbf{r}_b)_e$, interception time t_e, and aiming angles $[(\overline{\theta}_0)_i,(\overline{\psi}_0)_i]$. The reader should note that this picking of a set of sample values for the aiming angles is accomplished without ever knowing the explicit form of the PDF $B(\overline{\theta}_0,\overline{\psi}_0)$.

5. Pick a set of values for the initial direction (Θ_0,Ψ_0) of the bullet as it leaves the gun by sampling $C[\theta_0,\psi_0|(\overline{\theta}_0)_i,(\overline{\psi}_0)_i] \sin \theta_0$. These sample values are identified as $[(\theta_0)_i,(\psi_0)_i]$. Procedures for picking sample values for a random variable from its PDF are discussed by Shreider.[7] These sample values plus the fact that the birth site of each bullet is located at the origin of the GCS

[7]N.P. Buslenko, D.I. Golenko, Yu.A. Shreider, I.M. Sobol', and V.G. Sragovich, The Monte Carlo Method, The Method of Statistical Trials, Yu.A. Shreider, Editor, Translated from the Russian by G.J. Tee, Translation edited by D.M. Parkyn, Pergamon Press, Aylesbury, Bucks, Great Britain, 1967.

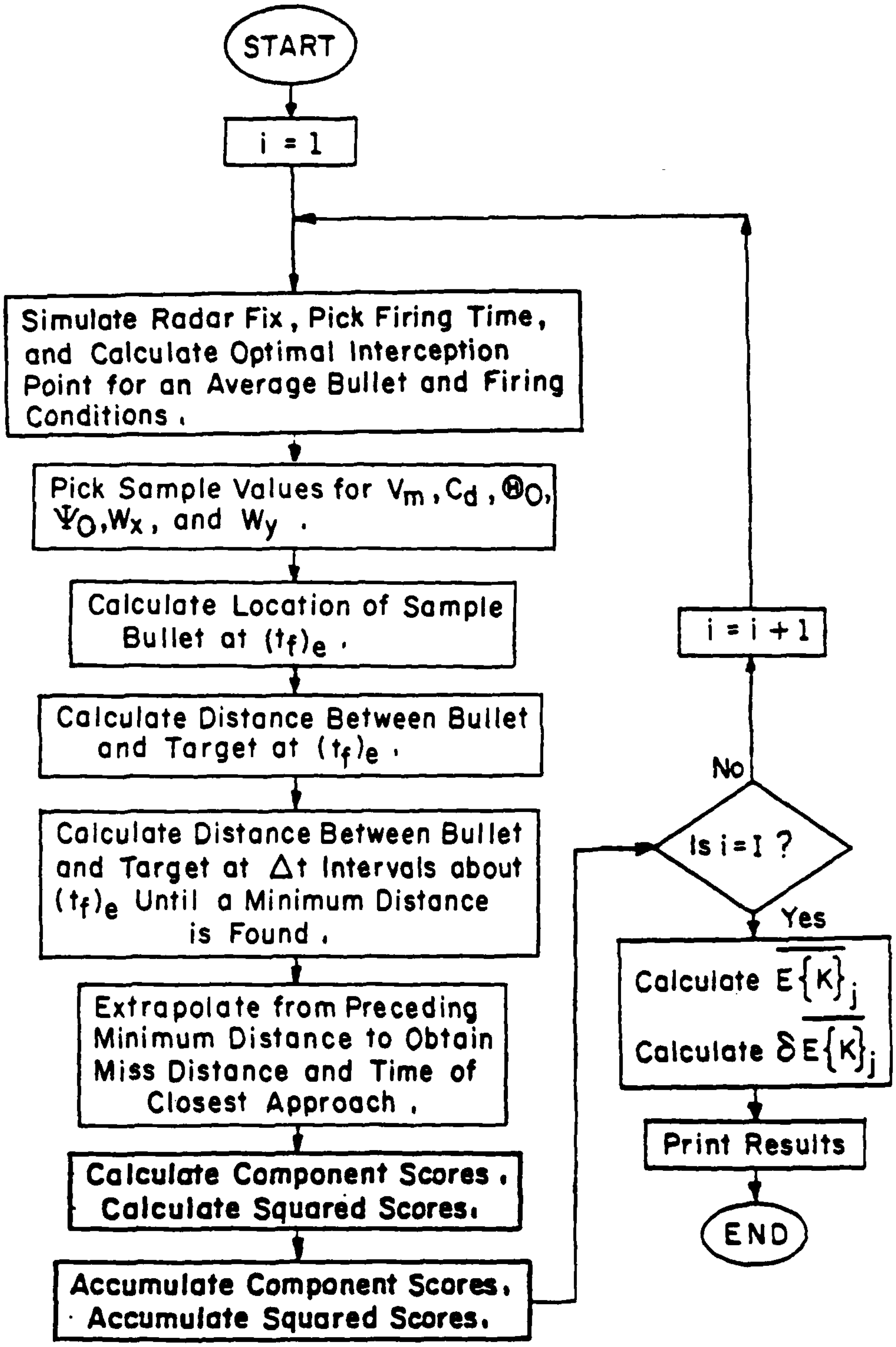

Figure 6. The Forward, or "Natural", Monte Carlo Estimation of the Expected-Value-of-Kill of a Critical Component in a Target Aircraft

provides the initial conditions for the bullet flyout.

6. Pick a value for each of the random variables (V_m, C_d, W_x, W_y) by sampling the appropriate PDF. These sample values are identified as $[(v_m)_i, (c_d)_i, (w_x)_i, (w_y)_i]$. These values are used below to calculate the necessary points on the trajectory of the sample bullet.

7. Simulate the firing of the sample bullet and calculate its true location $(\mathbf{r}_b)_e$ at the predicted time of interception t_e. The location of a bullet which has been in flight for a time $(t_f)_e$ is calculated by applying the differential corrections (Equations 4B-4D) to the line of sight (LOS) range vector $[(\theta_0)_i, (\psi_0)_i, l_e]$ (Equation 4A).

8. Determine the true location $(\mathbf{r}_a)_e$ and orientation $(\omega_a)_e$ (Figure 2) of the target aircraft at time t_e.

9. Reiterate Step 8 at selected intervals of Δt about t_e until an intersection with S_ς is bracketed (if such occurs). Set the kill-probability score to 0 and go to Step 14 if the bullet does not impact S_ς.

10. Using the bracketing data acquired in Step 9, interpolate to find an adequate approximation of the bullet intersection of S_ς. The location of the bullet at this point in its trajectory is identified by the position vector $(\mathbf{r}_b)_\varsigma$ and the associated time is t_ς. The bracketing data acquired in Step 9 can also be used to calculate an adequate approximation of the bullet velocity at time t_ς if this quantity is needed for the calculation of kill probabilities. Continue to Step 11 for a PD bullet, otherwise go to Step 12 for a PX bullet.

11. Calculate the kill probability for each critical component and then go to Step 13. Each kill-probability score is identified as λ_{ij} where j is the running index over critical components and J is the number of critical components in the target aircraft.

12. Determine the time when a PX bullet impacts the fuzing region, and start transporting the bullet into this region. Track the bullet until either a fuzing action occurs, the bullet impacts the target aircraft, or the bullet leaves the fuzing region. The burst of metal fragments produced by a detonation, if such an event occurs, must then be transported into the target and the kill probabilities of impacted critical components must be determined. A detailed description of such calculations is beyond the intent of this study and will not be addressed here. A possible approach is to

extend the stochastic description of the vulnerability process suggested by Beverly[8,9] for critical components exposed to spall fragments in heavily armored land vehicles (tanks) to include the higher target velocities associated with fixed-wing aircraft. We have not attempted to specify the detail with which the target aircraft is described but have merely assumed here and in Step 11 that it is adequately described.

13. Accumulate each score in a bin reserved for the associated component. Square each score and accumulate each squared score in another bin reserved for the associated component.

14. Reiterate Steps 1-13 until the histories for a total of I sample bullets have been simulated.

15. Applying the Law of Large Numbers[2], calculate the expected value of kill for each critical component as

$$\overline{E\{K\}}_j = \frac{1}{I}\sum_{i=1}^{I}\lambda_{ij} = \bar{\lambda}_j. \tag{9A}$$

16. Calculate an estimate $\delta\overline{E\{K\}}_j$ of the standard deviation of the estimated expected value of kill for each component as[10]

$$\delta\overline{E\{K\}}_j = \left[\frac{\sum_{i=1}^{I}\lambda_{ij}^2 - I\bar{\lambda}_j^2}{I(I-1)}\right]^{\frac{1}{2}}. \tag{9B}$$

Applying the Central Limit Theorem[2], this standard deviation can be interpreted as implying that approximately 68 percent of a large number of similar expected-value-of-kill estimates will fall within one standard deviation of the associated universe expected value.

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[8]W.B. Beverly, "A Stochastic Representation of the Vulnerability of a Critical Component in a Military Vehicle to Metal Fragments," Journal of Ballistics, Vol. 5, No. 4, October, 1981.

[9]W.B. Beverly, "The Application of the Monte Carlo Method to the Solution of the Internal Point Burst Vehicle Ballistic Model," Ballistic Research Laboratory Technical Report No. 02353, August 1981, (UNCLASSIFIED).

[10]Y.Beers, Introduction to the Theory of Error, Addison-Wesley Publishing Company, Inc., Reading, Mass., 1962.

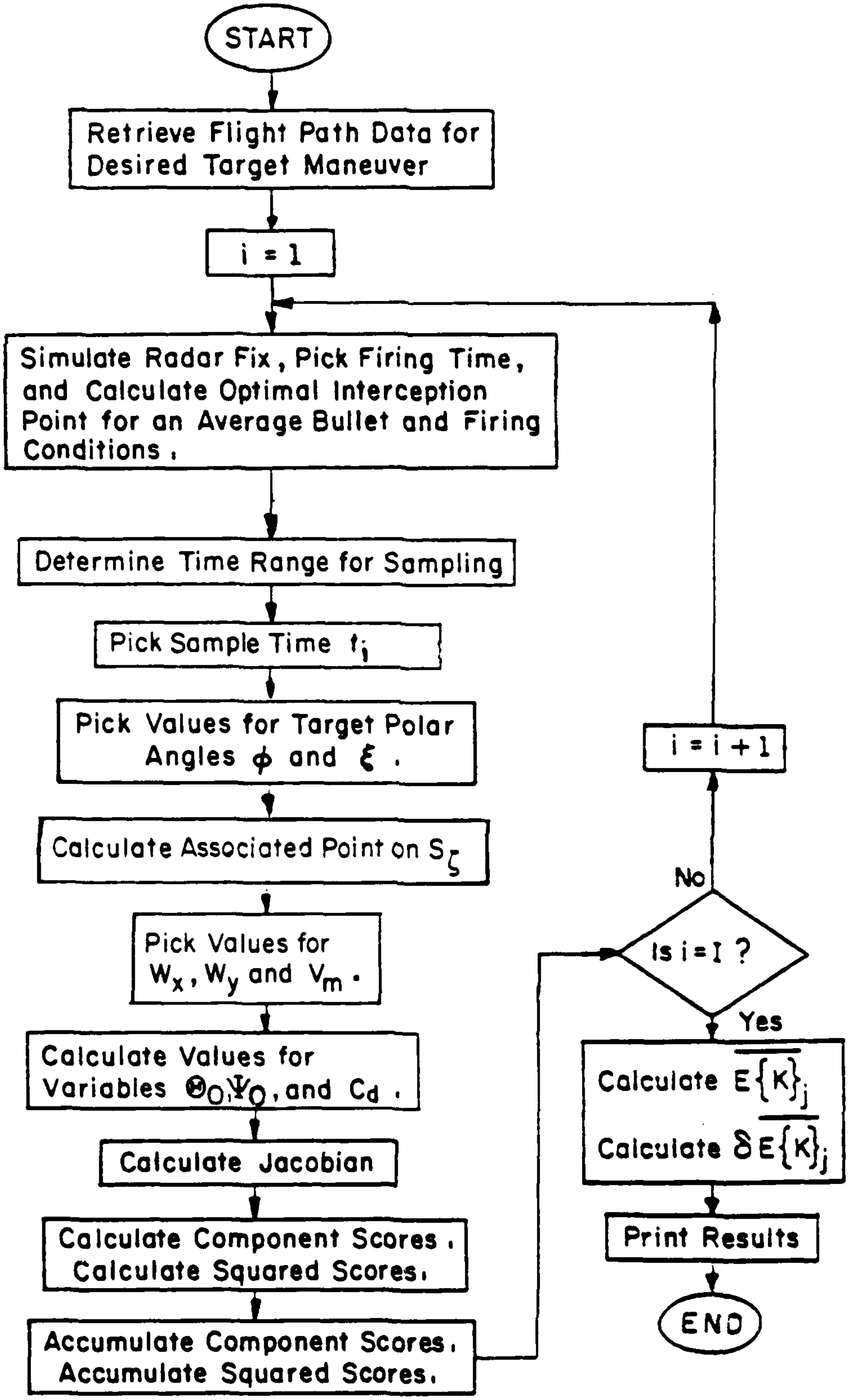

Figure 7. The Adjoint Monte Carlo Estimation of the Expected-Value-of-Kill of a Critical Component in a Target Aircraft

Step 16 completes the outline of a forward Monte Carlo solution of the expected-value-of-kill integral used in Equation 2. The alternate adjoint formulation given by Equation 7A is in a form such that sample bullets which usually impact S_ζ can be easily picked. A general Monte Carlo procedure for evaluating the adjoint integral is composed of the following steps (Figure 7).

1. Select the type of aircraft maneuver for which the expected value of kill is desired, and retrieve the predetermined flight path for this type of maneuver.

2. Simulate radar fixes on the target aircraft at selected times. The location, velocity, and acceleration of the target are either measured or calculated at each of these times.

3. Use the data derived in Step 2 to calculate optimal predictions of future target locations. Applying some preselected firing doctrine, pick a value for the firing time T_0 by sampling $A(t_0)$. This firing time is identified as $(t_0)_i$ where i is the sample index, and I is the number of sample histories to be used in the Monte Carlo calculation.

4. Using the data derived in Step 3 and the mean values of (V_m, C_d, W_x, W_y), calculate values for the aiming angles $(\bar{\Theta}_0, \bar{\Psi}_0)$. These calculated values are identified as $[(\bar{\theta}_0)_i, (\bar{\psi}_0)_i]$.

5. Pick a set of sample values $[(v_m)_i, (w_x)_i, (w_y)_i]$ for (V_m, W_x, W_y) by sampling the associated PDF's.

6. We now introduce the first step in the adjoint evaluation which differs from the forward evaluation just outlined. The range of t, and correspondingly $(t_f)_\zeta$, over which the major fraction of the bullets will impact S_ζ must be estimated by using the data derived in the first five steps. If Gaussian distributions are used to predict the uncertainties, non-zero probability tails will extend out to infinite values so that small, but non-zero, probabilities will exist for implausible firing or bullet-flyout conditions. In practice, these tails should be truncated and a reasonable target-time range $(R_t)_i$ be determined which will include only plausible events. The width of this sampling time range is identified at $(\Delta T)_i$, and will be used as a factor in the calculation of the event score.

7. Pick a sample value t_i for the target time. We will assume here that the sample value is picked with equal probability at any point on $(R_t)_i$. Using the notational convention adopted earlier, the associated time of flight to a bullet impact with S_ς would be identified as $[(t_f)_\varsigma]_i$. However, for the sake of simplicity, we will take the quantity $(t_f)_i$ in the remainder of the adjoint procedure to be the time of flight of the i^{th} sample bullet to its impact with S_ς.

8. Locate the target aircraft at t_i and determine in the gun coordinate system its velocity $\dot{\mathbf{r}}_a(t_i)$ and orientation $\omega_a(t_i)$. Use this data to locate the spherical region $(R_\varsigma)_i$ and its associated center $[(x_a)_\varsigma,(y_a)_\varsigma,(z_a)_\varsigma]_i$ in the GCS. If practical, determine the boundary of the associated S_ς as that part of the outer boundary of $(R_\varsigma)_i$ which can be impacted by any plausible bullet trajectories having the variable sample values just picked in Steps 1-8.

9. We are now ready to pick a sample set of values for the target-coordinate-system angles (Φ,Ξ). If, as anticipated, the exact boundary of $(S_\varsigma)_i$ is difficult to derive or its shape introduces sampling difficulties, then the following procedure may be used:
(a) Determine the range $(R_\phi)_i$ of ϕ-values and the range $(R_\xi)_i$ of ξ-values where these ranges include all possible target hits.
(b) Using these ranges, calculate the associated widths $(\Delta_\phi)_i$ and $(\Delta_\xi)_i$. These widths will be used as factors in the calculation of the event score.
(c) Pick a sample value for Φ with equal probability at any point on $(R_\phi)_i$. Similarly, pick a sample value for Ξ with equal probability at any point on $(R_\xi)_i$. This set of values is identified as (ϕ_i,ξ_i).
(d) Convert the sample angles just picked to the associated point $[(x_b')_\varsigma,(y_b')_\varsigma,(z_b')_\varsigma]_i$ (TCS) on the surface of $(R_\varsigma)_i$.
(e) Determine if this point is located at the entrance or exit of the sample bullet trajectory into $(R_\varsigma)_i$.
(f) Set the score to 0 and go to Step 17 if the point is a trajectory exit. Proceed to Step 10 if the point is a trajectory entrance into $(R_\varsigma)_i$. By definition, $(S_\varsigma)_i$ includes only trajectory entrance points into $(R_\varsigma)_i$.

10. Calculate the associated set of values for (Θ_0,Ψ_0,C_d) by inserting the Siacci Equation

$$l = \frac{(v_m)_i\,(t_f)_i}{1 + c_d\sqrt{(v_m)_i\,(t_f)_i}}, \tag{4A}$$

and the relation,

$$t_i = (t_0)_i + (t_f)_i \tag{3B}$$

between temporal variables into Equations 6B-6D:

$$l \sin\theta_0 \cos\psi_0 + (w_x)_i \left[(t_f)_i - \frac{l}{(v_m)_i}\right] = [(x_b'\ )_c]_i + x_s(t_i)\ , \tag{6A}$$

$$l \sin\theta_0 \sin\psi_0 + (w_y)_i \left[(t_f)_i - \frac{l}{(v_m)_i}\right] = [(y_b'\ )_c]_i + y_s(t_i)\ , \tag{6B}$$

$$l \cos\theta_0 - \frac{g\,(t_f)_i^2}{6}\left[\frac{3 + c_d\,(t_f)_i\,\sqrt{(v_m)_i}}{1 + c_d\,(t_f)_i\,\sqrt{(v_m)_i}}\right] = [(z_b'\ )_c]_i + z_s(t_i)\ . \tag{6C}$$

The resulting set of simultaneous equations can be solved by using an iterative procedure such as Newton's method.[11] This set of values is identified as $[(\theta_0)_i,(\psi_0)_i,(c_d)_i]$.

11. Calculate the killing probability of the sample event as described earlier in this Section in the outline of the forward Monte Carlo solution. If the bullet velocity is needed to perform this calculation, then an adequate approximation can be obtained from the change in the bullet coordinates for a small increase (or decrease) in the time of flight $(t_f)_i$. This quantity for either case is identified as Q_i, and will be used as a factor in the calculation of the event score.

12. Calculate the quantity $C[(\theta_0)_i,(\psi_0)_i|(\bar\theta_0)_i,(\bar\psi_0)_i]\ Sin(\theta_0)_i$. This quantity is identified as $C_i \sin(\theta_0)_i$, and will be used as a factor in the calculation of the event score.

13. Calculate the value of the Jacobian $J_c[\phi_i,\xi_i,(t_f)_i]$ (Equation 7B) for the event where the necessary derivatives are derived from Equations 6A-6C. The time dependence of $\mathbf{r}_s$ is derived for the particular target maneuver used. This quantity is identified as J_i, and will be used as a factor in the calculation of the event score.

14. Calculate the value of the PDF $G[(c_d)_i]$ for the event. This quantity is identified as G_i, and will be used as a factor in the calculation of the event score.

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[11]Peter Henrici, Elements of Numerical Analysis, John Wiley and Sons, New York, N.Y., 1964.

15. Calculate an event score for each critical component. These scores, identified as λ_{ij}, are given by

$$\lambda_{ij} = \Delta_i T \, \Delta_i \phi \, \Delta_i \xi \, C_i \sin(\theta_0)_i \, G_i \, J_i \, Q_i. \tag{11}$$

16. Square each component score. Accumulate each score and each squared score in bins reserved for these operations.

17. Calculate the scores for a total of I similar sample events by reiterating Steps 1-16.

18. Applying the Law of Large Numbers[2], calculate an estimate of the expected value of kill for each critical component as the mean of the associated scores, that is

$$\overline{E\{K\}}_j = \frac{1}{I}\sum_{i=1}^{I} \lambda_{ij} = \bar{\lambda}_j. \tag{12A}$$

A bar is placed over a quantity to identify its mean estimate.

19. Calculate an estimate $\delta\overline{E\{K\}}_j$ of the standard deviation of the estimated expected value of kill for each component as[10]

$$\delta\overline{E\{K\}}_j = \left[\frac{\sum_{i=1}^{I} \lambda_{ij}^2 - I\bar{\lambda}_j^2}{I(I-1)}\right]^{\frac{1}{2}}. \tag{12B}$$

Applying the Central Limit Theorem[2], this standard deviation can be interpreted as implying that approximately 68 percent of a large number of similar estimates will fall within one standard deviation of the associated universe expected value.

Step 19 completes an outline of an adjoint Monte Carlo evaluation of the adjoint expected-value-of-kill integral. An example two-dimensional problem is described in the next section of this report and its forward- and adjoint-calculated answers are compared.

IV. THE SOLUTION OF A PEDAGOGICAL EXAMPLE AIR-DEFENSE-END-GAME PROBLEM USING FORWARD AND ADJOINT SAMPLING PROCEDURES

A. The Problem-Set Description

We have devised a two-dimensional problem (Figure 8) whose Monte Carlo solution illustrates in a simple manner the procedures outlined in the preceding section of this report. This problem is composed of the following parts:

1. A Gun Coordinate System (GCS) is defined for a gun and radar-tracking system where the muzzle of the gun is taken to be the origin. The location of the gun muzzle is assumed to be precisely known with respect to the radar so that all uncertainties associated with locating the target in the GCS are due to inaccuracies in the radar fixes. We will, depending on analytical convenience, use either the Cartesian variables (x,y) or the polar variables (θ,s) (Figure 9) as the coordinates of the bullet position vector $\mathbf{r}_b$ and the target-circle-center position vector $\mathbf{r}_a$ in the GCS.

2. A circular, two-dimensional target of radius a is moving in the positive y direction along the line $x_a = 2\sqrt{3}b$ with a constant velocity of $v_a = b/sec$ where b is a length (Figure 8). The target center is located at $(x_a = 2\sqrt{3}b,\ y_a = 0)$ at time $t = 0$. A Target Coordinate System (TCS) is defined whose origin is the the center of the target circle and whose variables are taken to be the polar coordinates (ϕ,s) (Figure 10).

3. The radar detection and tracking of the target is assumed to begin at $y_a = 0$ and to continue until a bullet is fired. These radar measured target locations and velocities are presumed to be used in some unspecified manner to predict future target locations which in turn are used to calculate the aiming angle for a time of firing t_0. In these calculations, the x-coordinate of the target location and the x-component of the target velocity are assumed to be precisely measured so that all radar predictions place the target center on the line $x_a = 2\sqrt{3}b$. Similarly, during the calculation, the acceleration of the target along its path is taken to be precisely zero.

4. A bullet is fired at some time t_0 between $t = 1$ sec and $t = 2$ sec where the PDF of firing times is given by the triangular distribution $f_1(t_0)$ (Figure 11a). The target center, y_0, at the time of firing is located at $(y_a)_0 = y_0 = v_a t_0$. The uncertainty in the y-direction associated with the

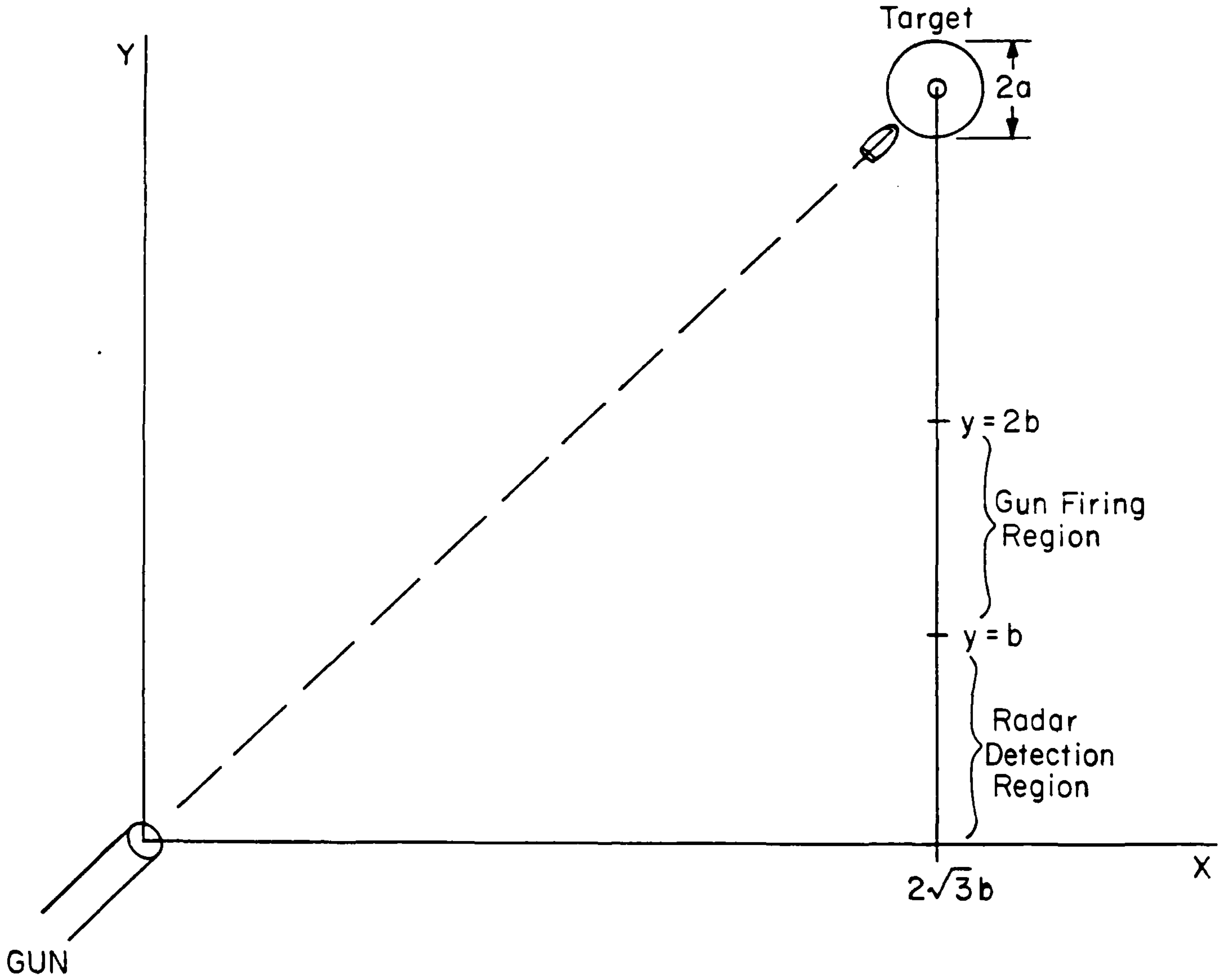

Figure 8. A Two-Dimensional Air-Defense End-Game Example Problem (Not to Scale)

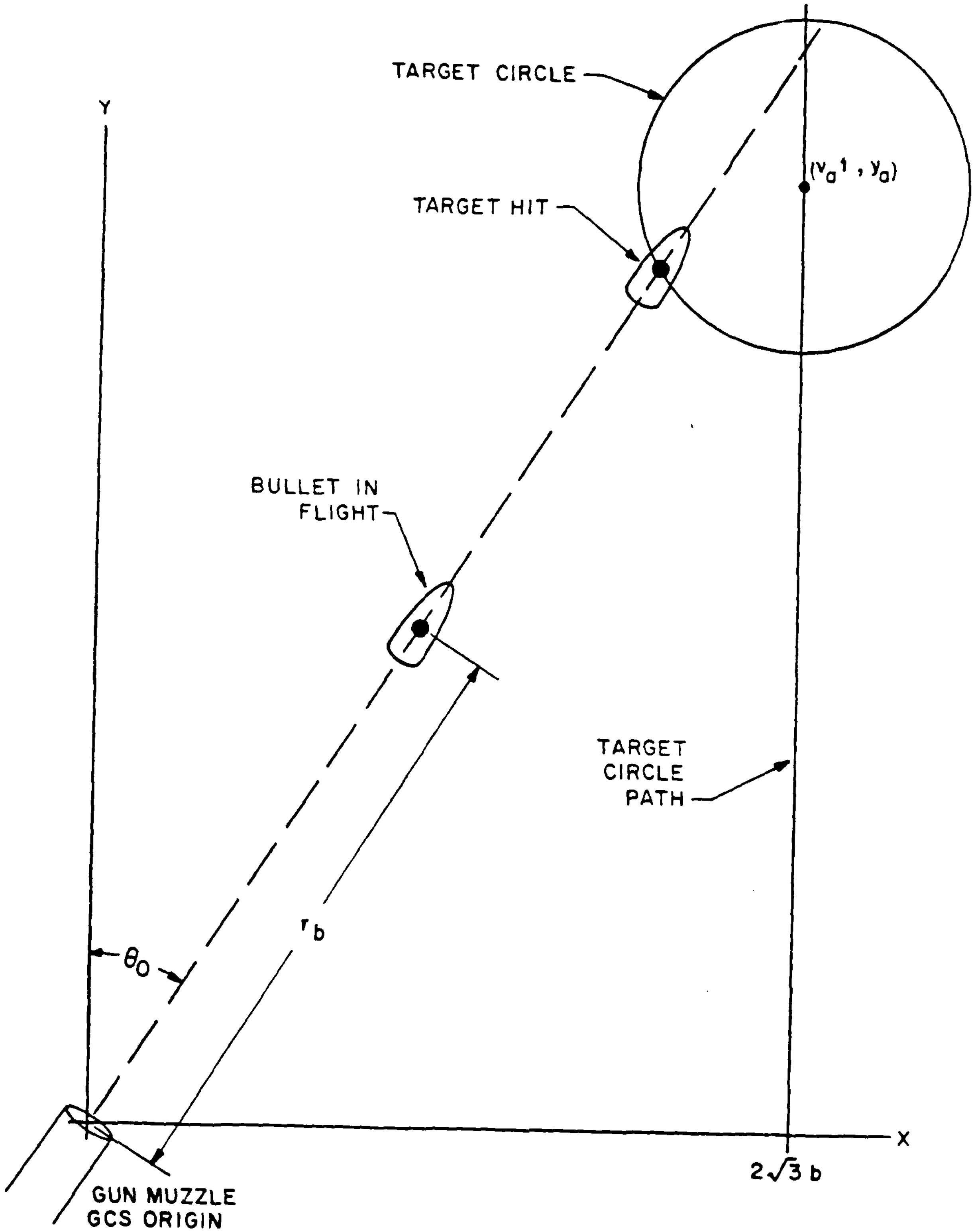

Figure 9. The Gun-Coordinate System (GCS) for the Example Problem (Not to Scale)

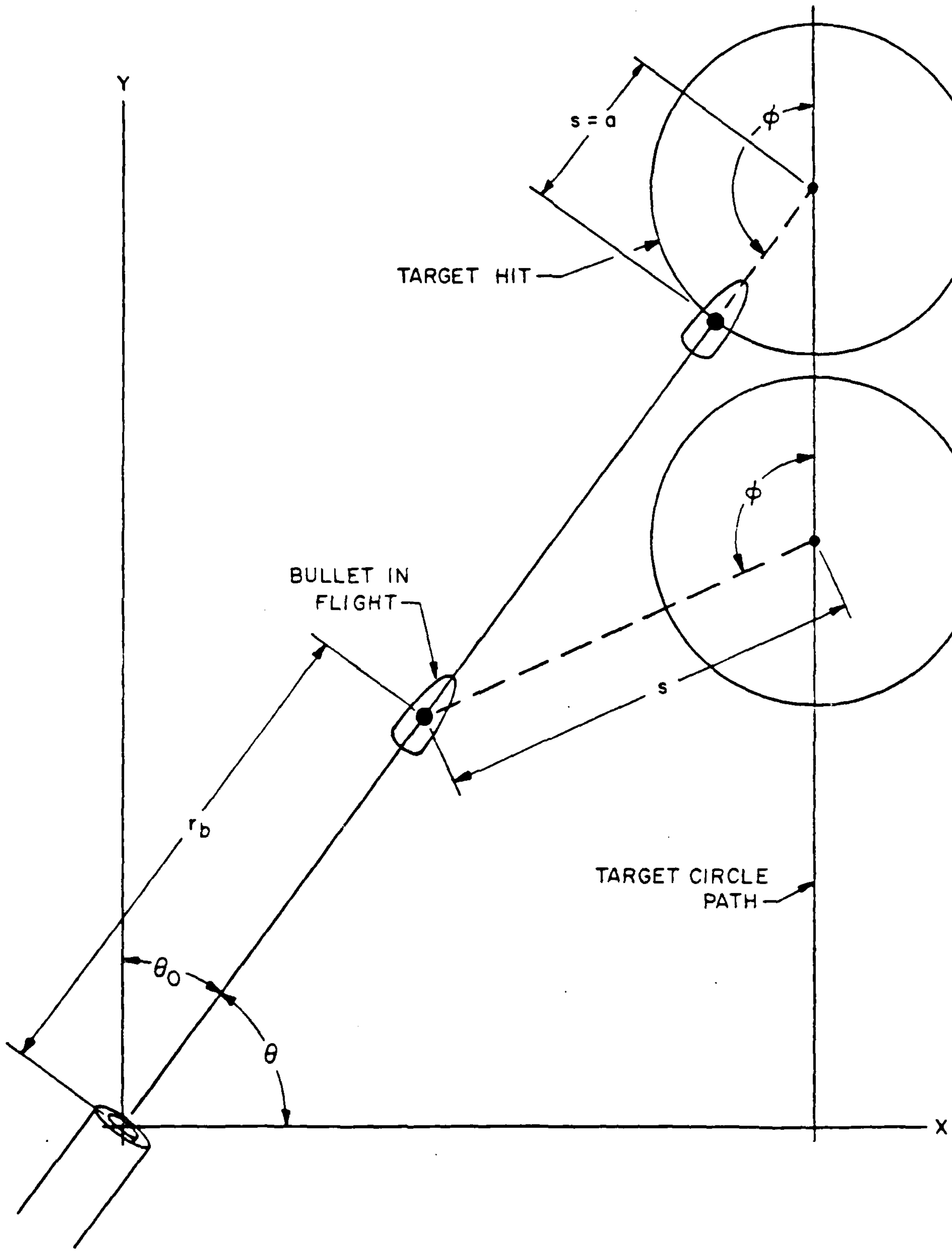

Figure 10. The Target-Coordinate System (TCS) for the Example Problem (Not to Scale)

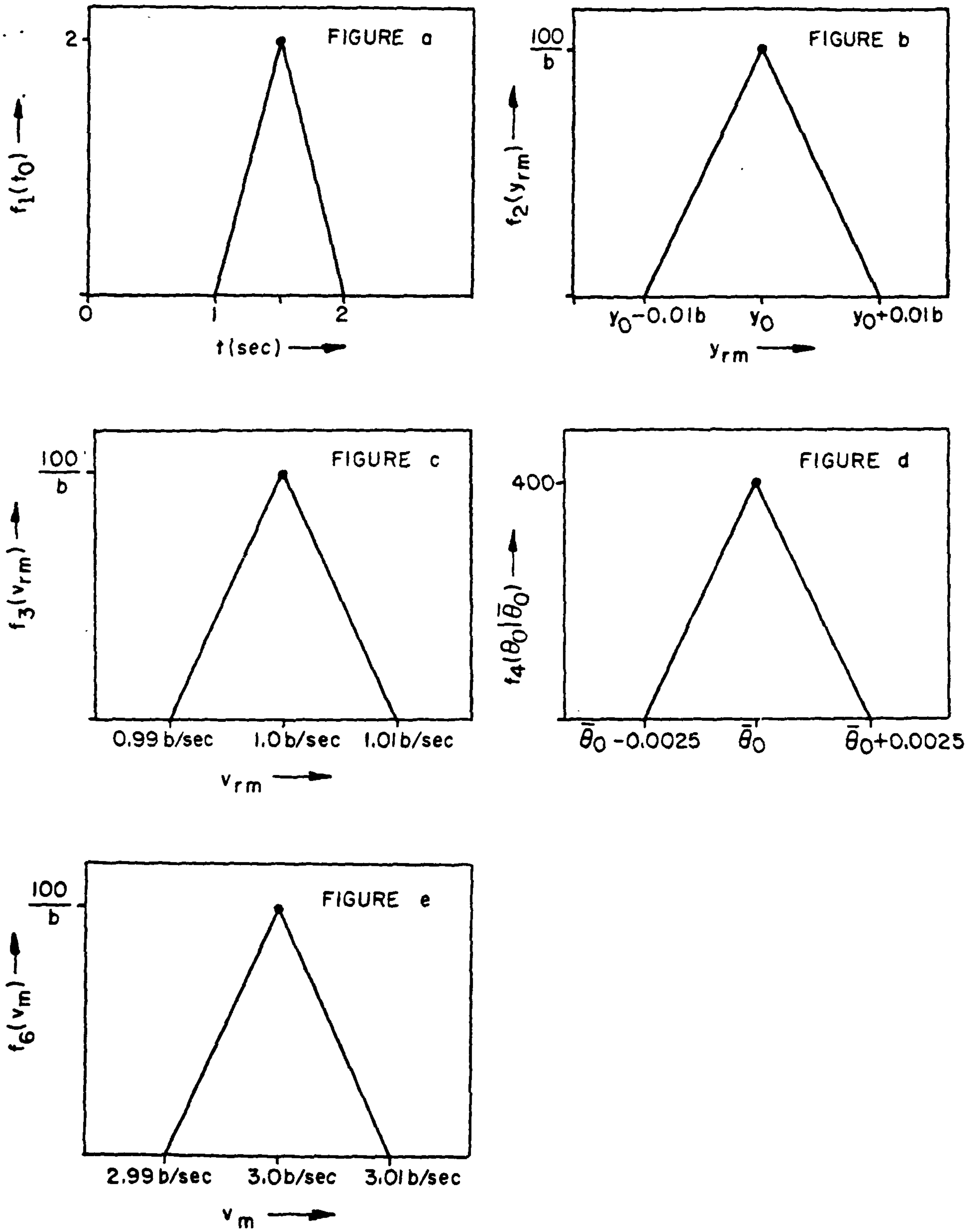

Figure 11. The Probability Density Functions Associated with the Example Problem

radar measured (rm) target-center location, y_{rm}, is identified at the time of firing, t_0, by the triangular conditional PDF, $f_2(y_{rm}|y_0)$ (Figure 11b), and the uncertainty in the y-direction associated with the radar measured target velocity v_{rm} is described by a similar triangular PDF $f_3(v_{rm})$ (Figure 11c).

5. The aiming angle $\overline{\theta}_0$ used for a firing event is derived from the measured values for target velocity and location and the mean value, $\overline{v}_m$, of the bullet muzzle velocity.

6. The true firing angle θ_0 due to the various uncertainties associated with positioning and firing the gun is predicted by the triangular conditional PDF $f_4(\theta_0|\overline{\theta}_0)$ (Figure 11d).

7. The true muzzle velocity v_m of the fired bullet has an uncertainty which is predicted by the triangular PDF $f_5(v_m)$ (Figure 11e). The quantity $\overline{v}_m$ is the mean muzzle velocity for a large number of similar firing events.

8. The bullet travels in a straight line with a velocity which decreases at a constant rate given by

$$v_b = v_m\,(1 - 0.2\ t_f), \tag{13A}$$

where t_f is the time that the bullet has been in flight. The distance $|\mathbf{r}_b|$ traveled by a bullet at a time of flight t_f is derived from Equation 12A as

$$|\mathbf{r}_b| = v_m\ t_f\,(1 - 0.1\ t_f). \tag{13B}$$

9. Associated with the target is a response function Q which in air-defense studies would quantify some specified loss of capability by a target component due to a hit. In this example problem, we assume that the loss of capability is a function only of the location of a hit on the target circle. This response function has the form, $Q_G[(\mathbf{r}_b)_H,(\mathbf{r}_s)_H]$, in the GCS and the simpler form, $Q_T[(\phi,a)]$, in the TCS. We identify the position vectors associated with target hits by affixing the subscript H. If Q_G and Q_T are unity for all hits, then the expected-value integral constructed below is merely the expected value of hit for the specified firing events.

Item 9 completes the description of the example problem. The "natural" formulation of the expected value $E\{\mathit{Target\ Response}\}$ of target response is given by the multiple integral

$E\{Target\ \ Response\} =$

$$\int \cdots_{R_G^H} \cdots \int f_1(t_0)\, f_2(y_{rm}|y_0)\, f_3(v_{rm})\, f_4(\theta_0|\overline{\theta}_0)\, f_5(v_m) \times$$
$$Q_G[(\mathbf{r}_b)_H,(\mathbf{r}_s)_H]\; dv_m\; d\theta_0\; dv_{rm}\; dy_{rm}\; dt_0 \tag{14}$$

where R_G^H is the region in the GCS space $(t_0,y_{rm},v_{rm},\theta_0,v_m)$ where bullets will hit the target circle.

B. The Forward Monte Carlo Estimation Procedures

The multiple integrals in Equation 14 can easily be evaluated using a "natural" Monte Carlo procedure by simulating a large number of firing events using the given PDF's and then calculating the mean target response. We outline below (Figure 12) such a procedure for calculating the expected value of hit $E\{H\}$ by taking $Q[(\mathbf{r}_b)_H,(\mathbf{r}_s)_H]$ to be 1 for all hits:

1. Pick a value for the firing time of the bullet by sampling the PDF $f_1(t_0)$. This sample value is identified as $(t_0)_i$ where i is the sample index and I is the number of Monte Carlo histories to be used in the calculation. Calculate the true target location at $(t_0)_i$ as $(y_0)_i = v_s\,(t_0)_i$.

2. Pick a value for the radar measurement of the target location at the time of firing by sampling the PDF $f_2[y_{rm}|(y_0)_i]$. As noted earlier, this sample value of the measured target location is always taken to lie on the true flight path with an uncertainty only in the y-coordinate. This sample value is identified as $(y_{rm})_i$.

3. Pick a value for the measured target velocity by sampling the PDF $f_3(v_{rm})$. This sample value is identified as $(v_{rm})_i$. As noted earlier, this sample value of the measured velocity is always taken to be in the positive y-direction.

4. Assuming that target locations at future times are derived from the sample values $(y_{rm})_i$ and $(v_{rm})_i$, calculate the aiming angle for which a bullet having the mean muzzle velocity $\overline{v}_m$ will hit the center of the target. This can be accomplished by inserting these sample values into the relation

$$t_f = \frac{[12b^2 + (y_{rm} + v_{rm}\, t_f)^2]^{\frac{1}{2}}}{\overline{v}_m\,(1 - 0.1\, t_f)} = g(t_f), \tag{15}$$

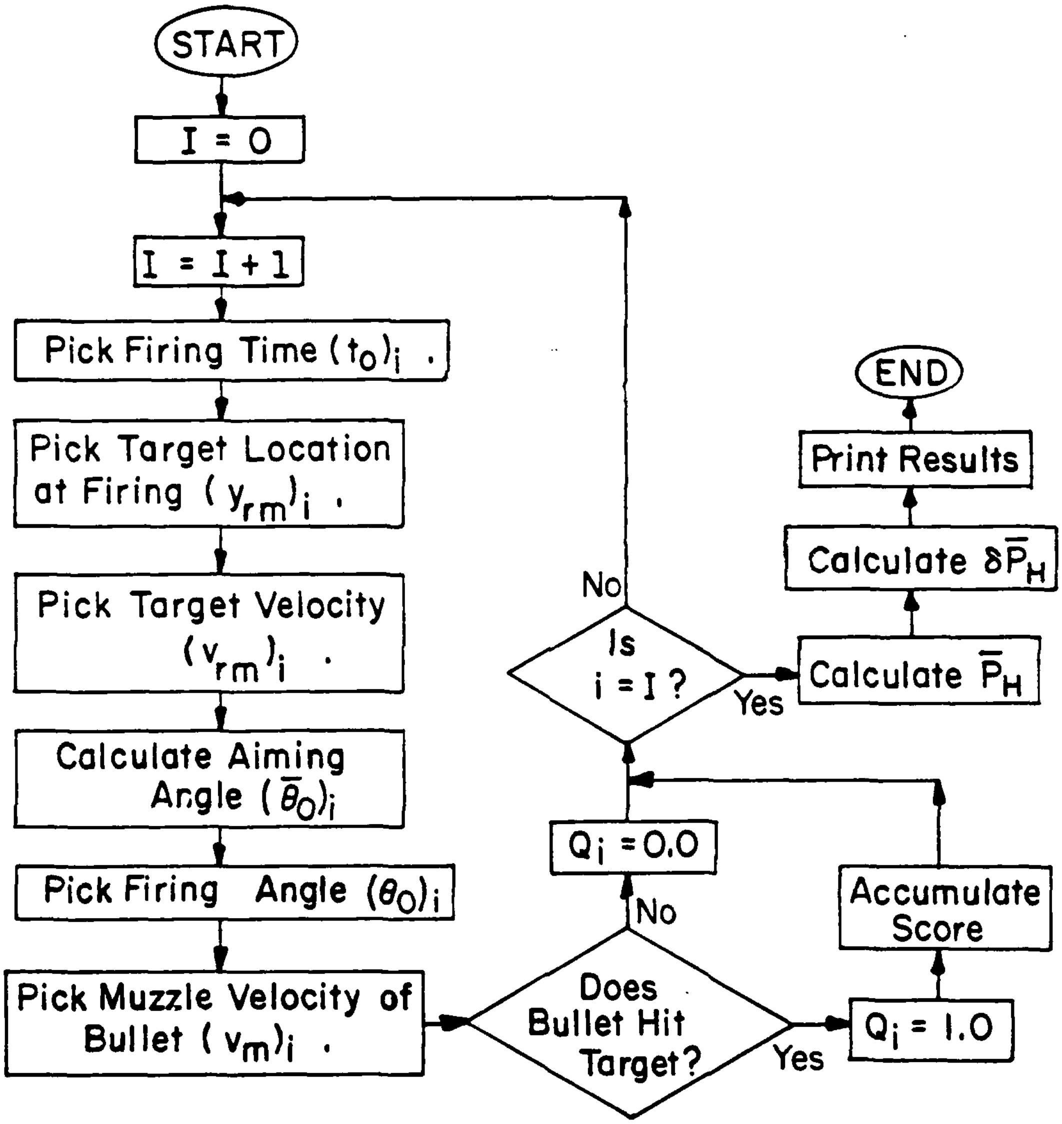

Figure 12. The Forward Monte Carlo Estimation of the Expected-Value-of-Hit for the Example Problem

derived from the time-of-flight analytic right triangle (Figure 13), and then iteratively locating the time of flight, $(t_f)_H$, to a hit as the fixed point[11] of $g(t_f)$. An aiming angle $(\bar{\theta}_0)_i$ is then derived for the anticipated location of the target at the time of hit.

5. Pick a value for the true firing angle by sampling the conditional PDF $f_4[\theta_0|(\bar{\theta}_0)_i]$. This sample value is identified as $(\theta_0)_i$. As noted earlier, the birth site of each bullet is taken to be the origin of the GCS.

6. Pick a value for the muzzle velocity of the fired bullet by sampling the PDF $f_5(v_m)$. This sample value is identified as $(v_m)_i$.

7. The determination of whether a bullet hits the target is not as easy as might be expected since both the target circle and bullet are moving. In the method used here, we first note that the square of the distance $M(t_f)$ between the bullet and the target-circle center is given by

$$M(t_f) = [x_b(t_f) - 2\sqrt{3}\ b]^2 + [y_b(t_f) - y_a(t)]^2, \tag{16}$$

where the bullet time of flight t_f is related to the target time t by

$$t = t_0 + t_f. \tag{17}$$

The time of flight at the distance of closest approach is given by setting the time derivative

$$\frac{dM(t_f)}{dt_f} = 0, \tag{18A}$$

and solving the resulting equation for its zero. Performing the indicated differentiation and grouping terms for convenience, the function $g(t_f)$ is derived such that

$$t_f = \frac{2\sqrt{3}\ b - H(t_f)}{v_m\ (1 - 0.1\ t_f)\ \sin\theta_0} = g(t_f) \tag{18B}$$

where

$$H(t_f) = \frac{[y_b(t_f) - y_a(t_f)]\ [v_m\ (1 - 0.2t_f)\ \cos\theta_0 - v_a]}{v_m\ (1 - 0.2t_f)\ \sin\theta_0}. \tag{18C}$$

The function $g(t_f)$ was determined by trial to be an acceptable iteration function for finding the fixed point of Equation 18B where the fixed point, $(t_f)_m$, is the time of flight at which the miss distance is a minimum.

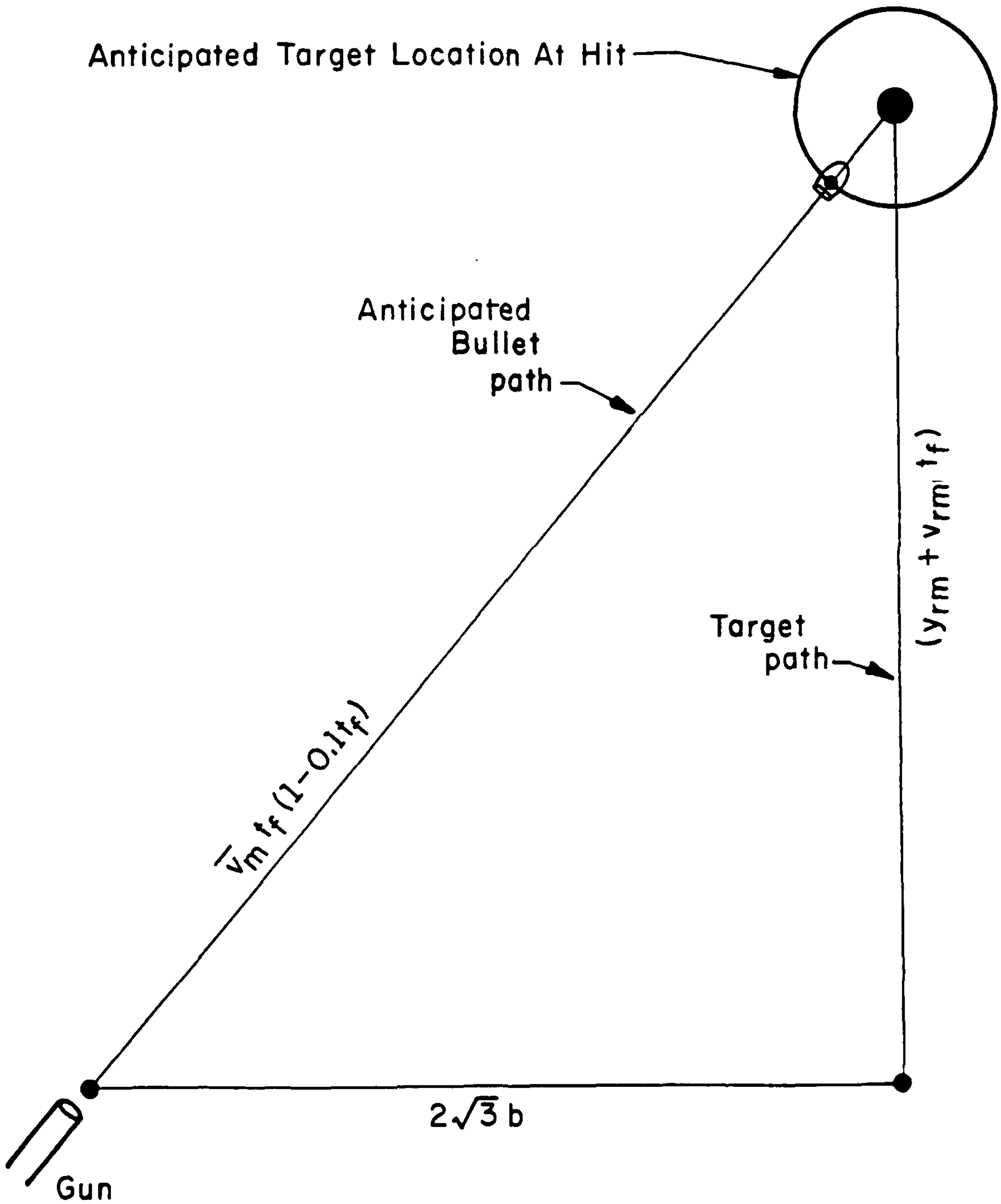

Figure 13. The Time-of-Flight Analytic Right Triangle Used To Calculate Aiming Angles (Not to Scale)

8. Calculate the location of the bullet and the target at time $(t_f)_m$. Assign a score of one to the sample event if the bullet is inside the target circle, otherwise assign a score of zero. The score is identified as Q_i.

9. Reiterate Steps 1-8 for a total of I histories.

10. Calculate the expected value for hitting the target as

$$E\{H\} = \bar{P}_H = \frac{1}{I}\sum_{i=1}^{I} Q_i. \tag{19A}$$

11. Calculate the standard deviation $\delta\bar{P}_H$ of $\bar{P}_H$ as

$$\delta\bar{P}_H = \left[\frac{\sum_{i=1}^{I} Q_i^2 - I\,\bar{P}_H^2}{I(I-1)}\right]^{\frac{1}{2}}. \tag{19B}$$

C. <u>The Adjoint Monte Carlo Estimation Procedures</u>

We will now attempt to change some of the variables of integration used in Equation 14 to obtain a form such that bullets which hit the target can always be picked as samples.

To accomplish this objective, we first derive a second analytic right triangle (Figure 14) which describes bullet intersections of the target circle. We then note that the pair of independent equations,

$$\tan\theta_0 = \frac{2\sqrt{3}b - a\sin\phi}{y_s(t) - a\cos\phi} = \frac{2\sqrt{3}b - a\sin\phi}{v_s t - a\cos\phi}, \tag{20A}$$

$$v_m = \frac{|\mathbf{r}_b|_H}{t_f(1-0.1t_f)} = \frac{[(2\sqrt{3}b - a\sin\phi)^2 + (v_s\,t - a\cos\phi)^2]^{\frac{1}{2}}}{t_f(1-0.1t_f)}, \tag{20B}$$

can be derived which relate (θ_0, v_m) in the GCS to (ϕ, t) in the TCS. The quantity, $|\mathbf{r}_b|_H$, in Equation 20B is the distance traveled by a bullet to a hit.

We assume that each realistic bullet trajectory (and target maneuver) for a particular set of values for $(t_0, \bar{\theta}_0)$ can be uniquely determined by either (θ_0, v_m) or (ϕ, t), so that all points in R_H^G space $(t_0, y_{rm}, v_{rm}, \theta_0, v_m)$ will transform one-to-one to points in R_H^T space, $(t_0, y_{rm}, v_{rm}, \phi, t)$. When this condition is

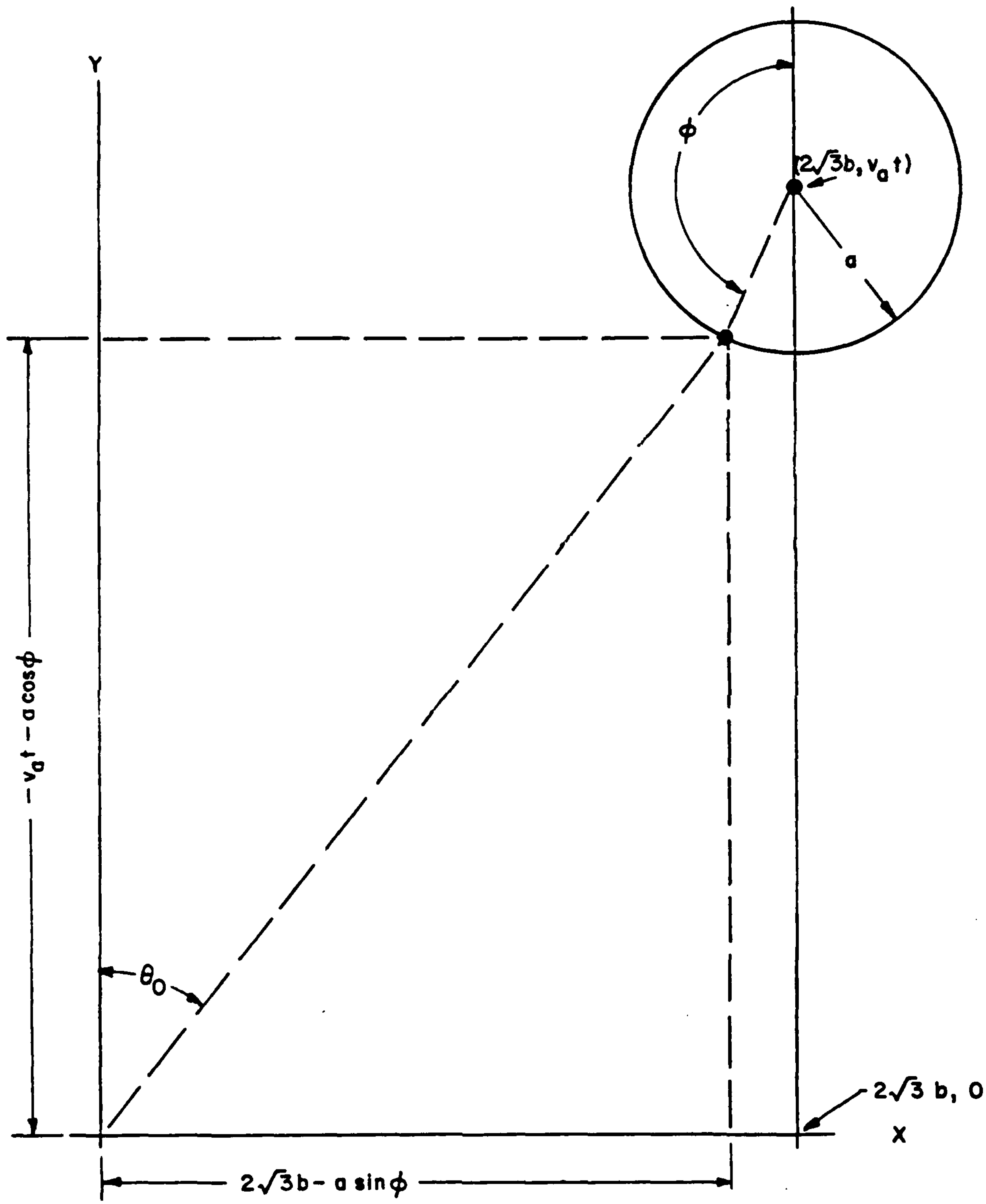

Figure 14. The Time-of-Flight Analytic Right Triangle Which Relates Target Hits in the GCS and the TCS (Not to Scale)

satisfied, the expected-value-of-target-response integral will also be given by

$$E\{\,TargetResponse\} =$$

$$\int\cdot\underset{R_H^T}{\cdot}\cdot\int f_1(t_0)\; f_2(y_{rm}|y_0)\; f_3(v_{rm})\; f_4(\theta_0|\overline{\theta}_0)\; f_5(v_m)\; J(\phi,t)\; Q_T(\phi,a)\; d\phi\; dt\; dv_{rm}\; dy_{rm}\; dt_0. \quad (21)$$

We have now described the target response in the target coordinate system by the new function $Q_T(\phi,a)$. The Jacobian of the transformation is the absolute value of the determinant of the indicated partial derivatives, that is

$$J(\phi,t)=\left|\frac{\partial(\theta_0,v_m)}{\partial(\phi,t)}\right| = \begin{vmatrix} \frac{\partial\theta_0}{\partial\phi} & \frac{\partial\theta_0}{\partial t} \\ \frac{\partial v_m}{\partial\phi} & \frac{\partial v_m}{\partial t} \end{vmatrix}. \quad (22)$$

The θ_0 partial derivatives in the preceding determinant can be derived from Equation 20A to be

$$\frac{\partial\theta_0}{\partial\phi} = \frac{a\,[y_a(t)\cos\phi + 2\sqrt{3}\sin\phi - a]\sin^2\theta_0}{[y_a(t) - a\cos\phi]^2}\,, \quad (23A)$$

$$\frac{\partial\theta_0}{\partial t} = \frac{v_a\,[2\sqrt{3}\,b - a\sin\phi]\sin^2\theta_0}{[y_a(t) - a\cos\phi]^2}. \quad (23B)$$

Similarly, the v_m partial derivatives can be derived from from Equation 20B to obtain

$$\frac{\partial v_m}{\partial\phi} = \frac{a\,[y_a(t)\sin\phi - 2\sqrt{3}b\cos\phi]}{(r_b)_h\, t\,(1 - 0.1t)}\,, \quad (23C)$$

and

$$\frac{\partial v_m}{\partial t} = \frac{t\,v_a\,[1 - 0.1t]\,[y_a(t) - a\cos\phi] - [(r_b)_h^2]\,[1 - 0.2t]}{[(r_b)_h]\,t^2\,[1 - 0.1t]^2}. \quad (23D)$$

The integrals in Equation 21 can also be evaluated by using the Monte Carlo method where sample events are now in part determined by the sample values picked for the new variables of integration ϕ and t. Such a procedure, where the first four steps are identical to those used earlier in the "natural" evaluation, is given below (Figure 15):

1. Pick a value for the firing time of the bullet by sampling the PDF $f_3(t_0)$. This sample value is identified as $(t_0)_i$. Calculate the location $(y_0)_i$ of the target center at the sample firing time.

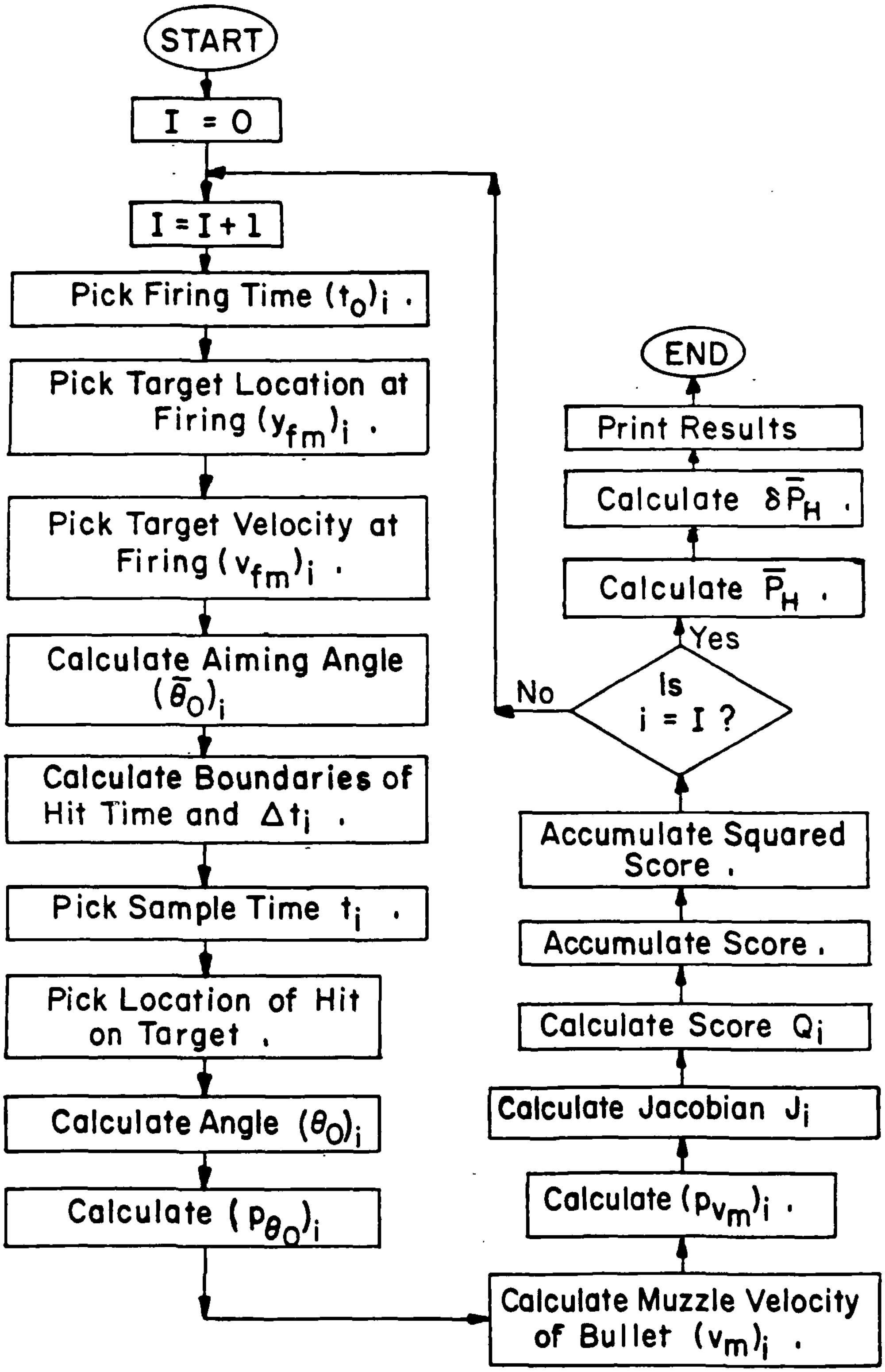

Figure 15. The Adjoint Monte Carlo Estimation of the Expected-Value-of-Hit for the Example Problem

2. Pick a value for the radar measurement of the target location at the time of firing by sampling the PDF $f_1[y_{rm}|(y_0)_i]$. This sample value is identified as $(y_{rm})_i$ where i is the sample index and I is the number of histories to be used in the calculation.

3. Pick a value for the measured target velocity by sampling the PDF $f_2(v_{rm})$. This sample value is identified as $(v_{rm})_i$.

4. Assuming that target locations at future times are derived from the sample values $(y_{rm})_i$ and $(v_{rm})_i$, calculate the aiming angle for which a bullet having the mean muzzle velocity $\bar{v}_m$ will hit the center of the target. This can be accomplished by inserting these sample values into the relation

$$t_f = \frac{[12b^2 + (y_{rm} + v_{rm}t_f)^2]^{\frac{1}{2}}}{\bar{v}_m\,(1 - 0.1t_f)} = g(t_f), \tag{15}$$

derived from the time-of-flight analytic right triangle (Figure 13), and then iteratively locating the time of flight, $(t_f)_H$, to a hit as the fixed point[11] of $g(t_f)$. An aiming angle $(\bar{\theta}_0)_i$ is then derived for the anticipated location of the target at the time of hit.

5. Step 4 is the last step in the adjoint-evaluation procedure which is the same as the forward-evaluation procedure described earlier in this Section. At this stage, the determination of the i^{th} event score, λ_i, could be regarded as the evaluation of the integral,

$$\lambda_i = \int\int f_4[\theta_0 \mid (\bar{\theta}_0)_i]\, f_5(v_m)\, J(\phi,t)\, Q_T(\phi,a)\, d\phi\, dt\,, \tag{24A}$$

where $\left[R^T_{\phi t}\right]_i$ is the region in the (ϕ,t) subspace which includes all points $[(t_0)_i,(y_{rm})_i,(v_{rm})_i,\phi,t]$ contained in R^T_H. If one wished, the integral could be evaluated using Monte Carlo procedures to a high precision as

$$\lambda_i = A_{\left[R^T_{\phi t}\right]_i} \frac{1}{N} \sum_{n=1}^{N} f_4[(\theta_0)_n \mid (\bar{\theta}_0)_i]\, f_5[(v_m)_n]\, J(\phi_n,t_n)\, Q_T(\phi_n,a)\,, \tag{24B}$$

where (ϕ_n,t_n) are picked with equal probability over $R^T_{\phi t}$, and $(\theta_0)_n$ and $(v_m)_n$ are calculated using (ϕ_n,t_n). The quantities n and N are introduced as a new running index and the number of associated samples, respectively, and $A_{\left[R^T_{\phi t}\right]_i}$ is the area of $\left[R^T_{\phi t}\right]_i$. However, applying the Law of Large Numbers[2], an acceptable i^{th} score can be derived using a single sample from the integral in Equation 24A, that is

$$\lambda_i = \left[A_{R^T_{\phi t}}\right]_i f_4[(\theta_0)_i \mid (\bar{\theta}_0)_i]\, f_5[(v_m)_i]\, J(\phi_n,t_n)\; Q_T(\phi_i,a) \,, \tag{24C}$$

where we have dropped the temporary index n and reintroduced the index i. We will describe the latter method although the average of five sample values was used in our actual calculations.

We would prefer to pick sample values for (ϕ,t) with equal probability at any point in $\left[R^T_{\phi t}\right]_i$, but its boundary may present calculational difficulties. Consequently, we will pick from a larger region, probably extending outside $\left[R^T_{\phi t}\right]_i$, which presents no calculational difficulty, and then discard all sample points $[(t_0)_i,(y_{rm})_i,(v_{rm})_i,\phi_i,t_i]$ which lie outside $\left[R^T_{\phi t}\right]_i$ by the simple expedient of setting to zero an appropriate factor which is used later to calculate the event score.

For the sample values $[(t_0)_i,(y_{rm})_i,(v_{rm})_i]$, just picked and the associated calculated aiming angle $(\bar{\theta}_0)_i$, determine the range $(R_t)_i$ of values for the target time which include all bullet-target intercepts associated with non-zero values for the PDF's $f_4[\theta_0|(\bar{\theta}_0)_i]$ and $f_5[(v_m)_i]$. The width of this range is identified as Δt_i and will be used as a factor in the calculation of the event score.

6. Pick a sample value of t with equal probability on the sampling range defined in Step 5. This sample time is identified as t_i.

7. Locate the target circle at t_i and calculate the range, $(R_\phi)_i$, (Figure 16) subtended by that part of the target circle which can be impacted by a bullet. The width of this range is identified as $\Delta\phi_i$ and will be used as a factor in the calculation of the event score.

8. Pick a value for ϕ over the range calculated in Step 7. This sample value is identified as ϕ_i.

9. Calculate in the GCS the location of the target center at time t_i. Calculate the location of the point on the target circle associated with (ϕ_i,t_i). The Cartesian coordinates of this point are identified as $[(x_H)_i,(y_H)_i]$. Since all bullet trajectories intersect the target circle, the quantity, $Q_T[(\phi_i),a]$, identified later as Q_i, is set to 1 for all events.

10. Calculate the value of θ_0 associated with $[(x_H)_i,(y_H)_i]$. This value is identified as $(\theta_0)_i$.

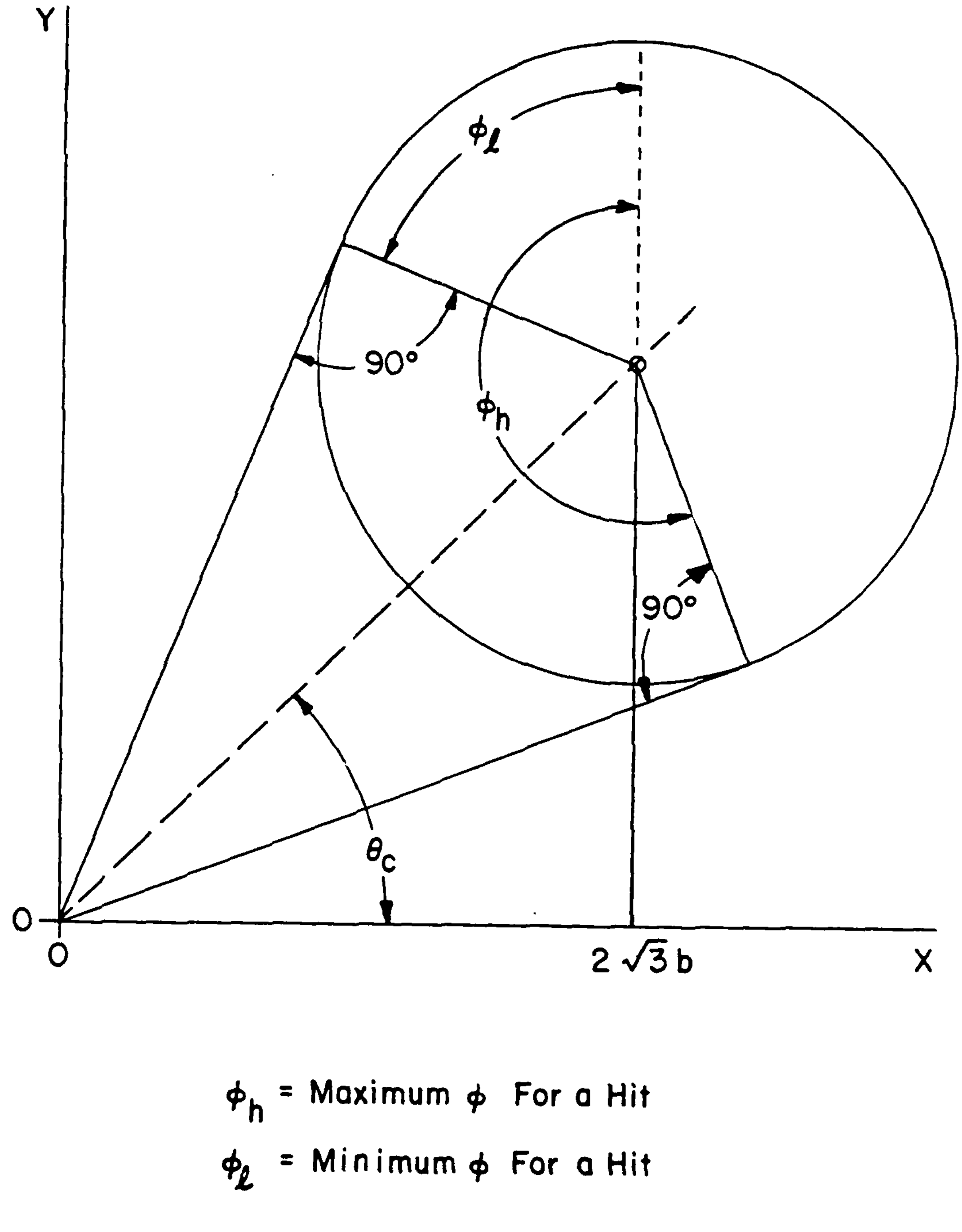

Figure 16. An Illustration of the Limits Associated with the Target Polar Angle ϕ (Not to Scale)

11. Calculate the value of the PDF $f_4[(\theta_0)_i|(\bar{\theta}_0)_i]$. This value is identified as $p_i(\theta_0)$ and will be used as a factor in the calculation of the event score. A zero value implies that the sample firing event lies outside R_H^T and should be discarded. Jump to Step 17 if $p_i(\theta_0) = 0$; otherwise, proceed to Step 12.

12. Calculate the value of v_m such that the sample firing will deliver a bullet at $[(x_H)_i,(y_H)_i]$ at time t_i. This value is identified as $(v_m)_i$.

13. Calculate the value of the PDF $f_5[(v_m)_i]$. This value is identified as $p_i(v_m)$ and will be used as a factor in the calculation of the event score. A zero value implies that the sample firing event lies outside R_H^T and should be discarded. Proceed to Step 17 if $p_i(v_m) = 0$; otherwise, proceed to Step 14.

14. Calculate the Jacobian for the history. This quantity is identified as J_i.

15. Calculate the event score λ_i as

$$\lambda_i = \Delta t_i \, \Delta\phi_i \, p_i(\theta_0) \, p_i(v_m) \, J_i \, Q_i. \tag{25}$$

Accumulate the score in a bin reserved for this operation.

16. Square the event score λ_i and accumulate the result in another bin reserved for this operation.

17. Reiterate Steps 1-16 for a total of I histories.

18. Applying the Law of Large Numbers[2], calculate an estimate of the probability of hitting the target as

$$\bar{P}_H = \frac{1}{I}\sum_{1=1}^{I}\lambda_i. \tag{26A}$$

19. Calculate an estimate $\delta\bar{P}_H$ of the standard deviation of the hit-probability estimate as[10]

$$\delta\bar{P}_H = \left[\frac{\sum_{i=1}^{I}\lambda_i^2 - I\,\overline{P_H}^2}{I(I-1)}\right]^{\frac{1}{2}}. \tag{26B}$$

Step 19 completes a general outline of the Monte Carlo evaluation of the multiple integrals used to formulate the expected value of hit for the example problem. We will discuss the answers to this problem below in Section IV.B.

C. The Problem Results

The FORTRAN computer programs FORWARD and BACK were constructed and used to calculate in turn forward and adjoint estimates of the expected-value-of-hit integrals given earlier in this section. A sufficient number of sample histories (5000) to obtain an acceptable standard deviation for the set of problems described below was used. In fact, the standard deviations obtained using this large number of sample histories are generally much smaller than those obtained in most air-defense end-game calculations. The calculational time needed by BACK was estimated to be approximately twice that of FORWARD for an equal number of sample histories.

Using each computer program, we calculated the expected value of hit for a set of targets whose radii varied systematically from $a = 0.0105b$ to $a = 0.0001b$ where b is taken to be 1000 (arbitrary units). The target velocity, v_a, was taken to be b/sec, and the mean bullet velocity $\bar{v}_b$, was taken to be 3b/sec. The fractional standard deviation percentage $\left[100\frac{\delta\bar{P}_H}{\bar{P}_H}\right]$ remained approximately constant for the set of answers calculated using BACK; while the fractional standard deviation percentage calculated using FORWARD increased with decreasing target-circle radius (and consequently decreasing P_H). This result was expected since a large fraction of the scores in the adjoint calculation have intermediate values while only 0's and 1's are scored in the forward mode. These answers are tabulated below in Table 1.

The desirability of using the adjoint method to calculate the expected value of hit where this value is small can be demonstrated by graphing the weighted standard deviations $W\frac{\delta\bar{P}_H}{\bar{P}_H}$ vs. a for the two calculational methods (Figure 17). The weight (calculational-time ratio) is taken to be 2 for the BACK computer program and 1 for the FORWARD computer program. The crossover point at $a = 3.4$ defines the point at which one should consider using adjoint sampling procedures.

	a	$\bar{P}_H$	$\delta\bar{P}_H$	$100\delta\bar{P}_H/\bar{P}_H$
TABLE A. THE FORWARD CALCULATION	10.5	0.61640	0.00688	1.116
	8.5	0.50660	0.00707	1.396
	6.5	0.40760	0.00695	1.705
	4.5	0.27840	0.00634	2.277
	2.5	0.16540	0.00525	3.177
	2.0	0.12560	0.00469	3.732
	1.5	0.10420	0.00432	4.147
	1.0	0.06420	0.00347	5.400
	0.5	0.03400	0.00256	7.539
	0.4	0.02520	0.00222	8.797
	0.3	0.02140	0.00205	9.564
	0.2	0.01320	0.00161	12.229
	0.1	0.00560	0.00106	18.847
TABLE B. THE ADJOINT CALCULATION	10.5	0.62371	0.00926	1.484
	8.5	0.53351	0.00753	1.412
	6.5	0.41915	0.00584	1.392
	4.5	0.29440	0.00400	1.359
	2.5	0.16662	0.00223	1.339
	2.0	0.13252	0.00180	1.356
	1.5	0.09870	0.00135	1.372
	1.0	0.06539	0.00090	1.372
	0.5	0.03222	0.00044	1.378
	0.4	0.02532	0.00035	1.396
	0.3	0.01977	0.00027	1.377
	0.2	0.01315	0.00018	1.351
	0.1	0.00640	0.00009	1.382

Table 1. The Example-Problem Answers Calculated Using Both Forward and Adjoint Procedures

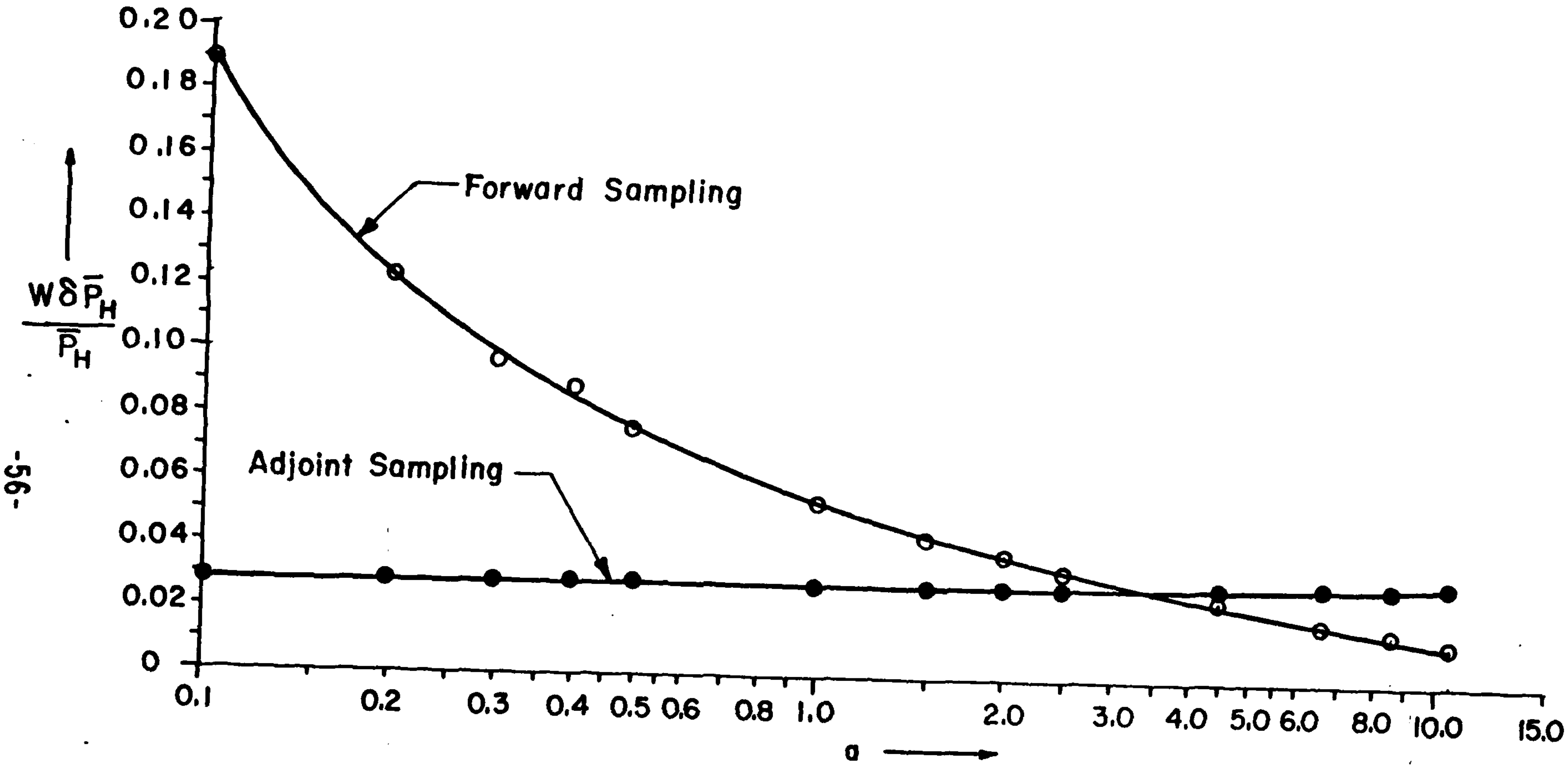

Figure 17. A Graphic Comparison of the Calculational Efficiencies of the Forward and Adjoint Monte Carlo Procedures

V. CONCLUSIONS AND PROGNOSIS

We have constructed an integral representation of the expected value of kill of a critical component in an aircraft by a bullet fired from a gun located on the ground, and have outlined a Monte Carlo evaluation of this integral where sample events are picked from their "natural" distributions. We then changed the variables of integration (θ_0,ψ_0,c_d) (firing angles and air-drag coefficient) used in the "natural" form of the expected-value-of-kill integral to (ϕ,ξ,t) (polar and azimuth angle in the target spherical coordinate system plus the target time), and outlined the Monte Carlo evaluation of the transformed integral. The necessary probability density functions for picking sample values for the variables of integration are identified and the bullet flyout is described by equations given in Reference 1. However, the outlined procedures are not restricted to any particular description of bullet flyout, but should be applicable to other methods of predicting the bullet flyout.

A quantitative comparison of the calculational efficiency of the two methods for a particular air-defense problem can only be obtained by trial calculations. In conducting a calculation, the effort of promenading a bullet near the target to obtain the interaction point would be expected to compare with the effort needed to calculate the values of the changed variables of integration by solving three simultaneous equations. However, as illustrated for the set of example problems in Section IV, the fractional standard deviation of a hit (or kill) probability estimate would be expected to increase with decreasing hit probability when forward sampling is used, and to remain approximately constant with decreasing hit probability when adjoint sampling is used. If the forward method is more efficient for problems having a hit probability near one, then a crossover exists at some hit probability beyond which the adjoint method would become the more efficient method. To determine when adjoint sampling is the more efficient method, the answers should be calculated for a set of representative air-defense-end-game problems similar to the example set described in Section IV.

Our complete omission of geometrical detail in the target when describing the interaction of bullets or shrapnel with its critical components has been deliberate. A combinatorial geometry[12] description of military

[12] W. Guber, R. Nagel, R. Goldstein, P.S. Mittelman, and M. Kalos, "A Geometric Description Technique for Computer Analysis of Both the Nuclear and Conventional Vulnerability of Armored Military Vehicles," Mathematical Applications Group, Inc., Report No. MAGI-6701, August 1967.

vehicles can be constructed to a high precision,[13 14] but the validity of the present procedures[15] being used to transport penetrators into surface targets and the prediction[5] of the incapacitation of an impact with a critical component have not, to the knowledge of the author, been rigorously verified. In the meantime, we expect that greatly simplified target descriptions, such as those used by the current MGEM[1] computer program provide all the target detail that is warranted.

A causal description of the detonation of a PX bullet and the transport of the resulting shrapnel into the target aircraft to possible impacts with critical components would require a prohibitive effort. A stochastic approach has been suggested in Reference 8 whereby the behavior of a large aggregate of similar ballistic events may be adequately describable by probability density functions of properly selected random variables. The existence of such a set of random variables has not been verified, but the adequacy criterion would not be expected to be very severe if the majority of the metal fragments impacting a critical component have perforated only one or two solid barriers. In any event, the adequate description of such ballistic effects is essential to vulnerability analysises, and a serious attempt should be made to develop and verify such a predictive model.

If such a stochastic description of ballistic events is developed and verified, then calculational methods developed for radiation transport[16 17]

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[13]L. Bain, and M. Reisinger, "The GIFT Code User Manual; Vol. I, Introduction and Input Requirements," Ballistic Research Laboratory Technical Report No. 1902, July 1975, (UNCLASSIFIED).

[14]G. Kuehl, M. Reisinger, and L. Bain, "The GIFT Code User Manual; Vol. II, The Output Options," Ballistic Research Laboratory Technical Report No. 02189, September 1979, (UNCLASSIFIED).

[15]T.S. Hafer and A.S. Hafer, "Vulnerability Analysis for Surface Targets (VAST): An Internal Point Burst Model," Ballistic Research Laboratory Technical Report No. 02154, April 1979, (UNCLASSIFIED).

[16]J. Spanier and E.M. Gelbard, Monte Carlo Principles and Neutron Transport Problems, Addison-Wesley Publishing Company, Reading, Massachusetts, Menlo Park, California, London, Don Mills, Ontario.

[17]W.C. Roesch, "Mathematical Theory of Radiation Fields," Basic Concepts of Dosimetry, F.H. Attix and W.C. Roesch, Editors, Volume I, Fundamentals, Second Edition, Academic Press, New York, San Francisco, and London, 1968.

may be advantageously applied to ballistic vulnerability studies. For example, looking at the shrapnel generated during the detonation of a PX bul!·t, we would expect that the formulation of the expected-value-of-kill integral would permit the calculation of the adjoint fragment fluence, $\phi(\mathbf{r}_0,\mathbf{w}_0,\mathbf{b}_0)$, over the fuzing region, R_f, for each critical component in the aircraft. The kill probability per fragment, P_K, would then be given as

$$P_K = \iiint_{R_f} S_f(\mathbf{r}_0,\mathbf{w}_0,\mathbf{b}_0)\, \phi(\mathbf{r}_0,\mathbf{w}_0,\mathbf{b}_0)\, d\mathbf{r}_0 d\mathbf{w}_0 d\mathbf{b}_0, \tag{27A}$$

Where $S_f(\mathbf{r}_0,\mathbf{w}_0,\mathbf{b}_0)$ is the probability density of shrapnel fragments produced during a fuzing action. The quantities, $(\mathbf{r}_0,\mathbf{w}_0)$, are the position vector and direction of motion, respectively, of a fragment in a target coordinate system, and $\mathbf{b}_0$ is a generalized set of variables which can be used to characterize fragments. In practice, $\phi(\mathbf{r}_0,\mathbf{w}_0,\mathbf{b}_0)$ would be evaluated in bins over the fuzing region, R_f, during a one-time calculation, and P_K would then be calculated as the sum of the products, $(S_f)_i\, \phi_i$, that is,

$$P_K = \sum_{i=1}^{I} (S_f)_i\, \phi_i\, . \tag{27B}$$

The quantity i is the running index over bins in the fuzing region about the target, and I is the total number of such bins.

The advantage of this method is that the adjoint fluence can be evaluated over the fuzing region of a plane in a one-time calculation and then be reused for any number of specific problems. A possible disadvantage in using the adjoint fluence is that its evaluation may be laborious and the dimensionality of the mesh needed for its description over the fuzing region may be large. In any event, the feasibility of such a procedure should be investigated.

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APPENDIX

THE MONTE CARLO EVALUATION OF EXPECTED-VALUE INTEGRALS

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The general features of a Monte Carlo evaluation of the expected-value-of-kill integrals used to describe the efficiency of air-defense gun systems are illustrated below by the triple integral

$$E\{Q(X,Y,Z)\} = \iiint_{R_{xyz}} S(x,y,z)\, Q(u,v,w)\, dx\, dy\, dz =$$

$$\iiint_{R_{xyz}} S(x,y,z)\, Q[u(x,y,z), v(x,y,z), w(x,y,z)]\, dx\, dy\, dz. \tag{A1}$$

The quantity $S(x,y,z)$ is the joint probability density of values for the set of random variables (X,Y,Z) in a region of interest R_{xyz} of xyz space, and $Q(u,v,w)$ is any well-behaved function with a finite maximum value. In certain physical problems, $S(x,y,z)$ is often identified as the source term, and $Q(u,v,w)$ is identified as a pay-off function or detector-response function. For example, $S(x,y,z)$ could describe the probability density of particles generated by some physical process, and $Q(u,v,w)$ could describe the interaction of these particles with a detector after their passage through an interactive medium. In our air-defense end games, the source term is taken to include the uncertainities associated with detecting and tracking a target, aiming the gun and firing the bullet, and transporting the bullet toward the target; and the pay-off function is the kill probability of a critical component in the target by the bullet. The source term will have its largest values in some known region and will become vanishingly small at known distances from this region. Therefore, the region R_{xyz} is generally taken to include all points in xyz space whose inclusion in an expected-value integral will make a significant contribution to the expected value. A source term is generally assumed to be normalized to unity unless otherwise noted, that is

$$\iiint_{R_{xyz}} S(x,y,z)\, dx\, dy\, dz = 1. \tag{A2}$$

According to Kaplan[15] the functions

$$u = u(x,y,z), \tag{A3}$$

$$v = v(x,y,z), \tag{A4}$$

and

[15] W. Kaplan, Advanced Calculus, Addison-Wesley Publishing Company, Inc., Cambridge 42, Mass, 1953.

$$w = w(x,y,z), \tag{A5}$$

are assumed to be defined and continuous in the region R_{xyz} of xyz space. The corresponding points (u,v,w) lie in the region R_{uvw} of uvw space, and it is assumed that the inverse relations

$$x = x(u,v,w), \tag{A6}$$

$$y = y(u,v,w), \tag{A7}$$

and

$$z = z(u,v,w), \tag{A8}$$

are defined and continuous in the region R_{uvw} of uvw space, so that the correspondence between R_{uvw} and R_{xyz} is one-to-one. As used in Equation A1, Equations A3-A5 causally describe some physical process. In our particle example, (X,Y,Z) are taken to quantify the essential features of particles at their generation (birth particles), and (U,V,W) describe these same particles (residual particles) at their interaction with the detector after passing through an interactive medium.

In practice, the expected-value integrals used in air-defense studies are often of much higher dimensionality than that used in the example triple integral. In such cases where a confidence level of 5% or so is acceptable, the Monte Carlo method is quite often the most efficient method of evaluation since convergence toward the true value always goes as $1/\sqrt{N}$ where N is the number of sample histories. An outline of a general Monte Carlo evaluation of the integral used in Equation 1 is given below:

1. Pick a set of values for (x,y,z) by sampling $S(x,y,z)$. This set of values is identified as (x_i,y_i,z_i) where i is the running index over sample events.

2. Calculate the associated values of (u,v,w) by using Equations A3-A5. This set of values is identified as (u_i,v_i,w_i). In a physical problem, this operation is often interpreted as the simulation of a physical event such as the transport of a particle through an interactive medium.

3. Calculate the score for the event as $Q(u_i,v_i,w_i)$. This quantity is identified as Q_i.

4. Accumulate Q_i in a bin reserved for this operation. Square the score and similarly accumulate the squared score in a second bin.

5. Reiterate Steps 1-4 until a total of I sample events have been simulated.

6. Applying the Law of Large Numbers[2], calculate an estimate of the triple integral in Equation A1 as the mean of the sample scores, that is

$$\overline{E\{Q(U,V,W)\}} = \overline{Q} = \frac{1}{I}\sum_{i=1}^{I} Q_i. \tag{A9}$$

The mean estimate of an integral is identified by placing a bar over its symbol.

7. Calculate an estimate of the confidence level of the preceding estimate as the standard deviation of the mean estimate, that is

$$\delta\overline{E\{Q(U,V,W)\}} = \delta\overline{Q} = \left[\frac{\sum_{i=1}^{I} Q_i^2 - I\overline{Q}^2}{I(I-1)}\right]^{\frac{1}{2}}. \tag{A10}$$

Applying the Central Limit Theorem[2], this estimate of the confidence level of the calculation can be interpreted as implying that approximately 68% of a large number of similar estimates will fall within one standard deviation of the true value of the triple integral.

Step 7 completes the outline of a general solution of the triple integral in Equation A1. This triple integral can be reformulated as

$$E\{Q(U,V,W)\} = CE\{S(X,Y,Z)\} =$$

$$C\iiint_{R_{uvw}} S[x(u,v,w),y(u,v,w),y(u,v,w)]\,J(u,v,w)\frac{Q(u,v,w)}{C}\,du\,dv\,dw \tag{A11}$$

by changing the variables of integration from (x,y,z) to (u,v,w). The quantity $J(u,v,w)$ is the Jacobian of the transformation, that is

$$J(u,v,w) = \left|\frac{\partial x,\partial y,\partial z}{\partial u,\partial v,\partial w}\right|. \tag{A12}$$

The quantity C is used to normalize the function $Q(u,v,w)$ to unity over this region, that is

$$\iiint_{R_{uvw}} Q(u,v,w)\,du\,dv\,dw = C. \tag{A13}$$

The transformed triple integral of Equation A11 can also be evaluated using the Monte Carlo method by interchanging the roles played by the functions $S(x,y,z)$ and $\frac{Q(u,v,w)}{C}$ in the Monte Carlo simulation procedure. Such a procedure is composed of the following steps:

1. Pick a set of values for (u,v,w) by sampling $\frac{Q(u,v,w)}{C}$ over the region R_{uvw}. This set of values is identified as (u_i,v_i,w_i) where i is the running index over sample events.

2. Calculate the associated values of (x,y,z) by using the set of inverse equations derived from Equations A3-A5. This set of values is identified as (x_i,y_i,z_i). In a physical problem, this operation can often be interpreted as the adjoint simulation of a physical event such as the transport of a particle from a detector back to its origin.

3. Calculate the score for the event as $CJ(u_i,v_i,w_i)S(x_i,y_i,z_i)$. This quantity is identified as CJ_iS_i or $C\lambda_i$.

4. Accumulate $C\lambda_i$ in a bin reserved for this operation. Square the score and similarly accumulate the squared score in a second bin.

5. Reiterate Steps 1-4 until a total of I sample events have been simulated.

6. Applying the Law of Large Numbers[2], calculate an estimate of the triple integral in Equation A11 (and Equation A1) as the mean of the sample scores, that is

$$\overline{E\{Q(U,V,W)\}} = \frac{C}{I}\sum_{i=1}^{I} J_i\, S_i = C\,\bar{\lambda} \tag{A14}$$

where

$$\bar{\lambda} = \sum_{i=1}^{I} J_i S_i. \tag{A15}$$

7. Calculate an estimate of the confidence level of the preceding estimate as the standard deviation of the mean estimate, that is

$$\delta\overline{E\{Q(U,V,W)\}} = \left[\frac{\sum_{i=1}^{I} C^2\lambda_i^2 - I\,C^2\bar{\lambda}^2}{I(I-1)}\right]^{\frac{1}{2}}. \tag{A16}$$

Applying the Central Limit Theorem[2], this estimate of the confidence level of the calculation can be interpreted as implying that approximately 68% of a large number of similar estimates will fall within one standard deviation of the true value of the triple integral.

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